

Abstract: In general relativity, the space we inhabit is modeled by a Riemannian manifold. The fundamental restriction this theory places upon spatial geometry is a lower bound on this manifold's scalar curvature. It is an important problem in pure geometry to understand the geometric and topological features of this condition. For instance, if a manifold has positive scalar curvature, what may we conclude about the lengths of its curves, the areas of its surfaces, and the topology of the underlying manifold? I will explain many results (originally proven by Schoen-Yau and Gromov-Lawson) in this direction, and sketch proofs by analyzing objects I call 'spacetime harmonic functions.' Leveraging these new ideas, I will also describe progress on geometric versions of the following questions: How flat is a gravitational system with little total mass? How can we tell when matter will coalesce to form a black hole?