We study locally harmonic maps between pseudo-Riemannian surfaces. These maps are classified by the classification of the solutions of a generalized sine-Gordon equation. We then study the one-soliton solutions of this equation and we find the corresponding harmonic maps in a unified way. Finally, we discuss a Backlund transformation of the harmonic map equations that provides a connection between the solutions of two sine-Gordon type equations.