On super-strong Wilf equivalence classes of permutations

A relatively new, but rapidly growing branch of Combinatorics, that has its roots in Theoretical Computer Science, is the study of *patterns in permutations and words*. In an attempt to study pattern avoidance using generating functions, Herbert Wilf defined the notion of *Wilf equivalence* for patterns, in general. We will present several notions of Wilf equivalence and other types of equivalences concerning *consecutive patterns* and, more specifically, *embeddings* in permutations and words, and we will then focus on *super-strong Wilf equivalence*.

Super-strong Wilf equivalence is a particular type of Wilf equivalence on words that was originally introduced as 'strong Wilf equivalence' by Kitaev et al. [Electron. J. Combin. 16(2)] in 2009. In the first part of the talk we provide a necessary and sufficient condition for two permutations in n letters to be super-strongly Wilf equivalent, using distances between letters within a permutation. In this way super-strong Wilf equivalence classes in the symmetric group $S_{\_n}$ on n letters are in bijection with *pyramidal sequences of consecutive differences*. This result helps us to settle partially – in the special case of super-strong Wilf equivalence - a conjecture in the aforementioned article by Kitaev et al., stating that the cardinality of each Wilf equivalence class of a permutation is a power of 2.

In the second part of the talk we enumerate all super-strong Wilf equivalence classes of S_n by giving a recursive formula in terms of a two-dimensional analogue of the sequence of the number of *non-interval permutations* in S_n. By a refined version of this formula we also enumerate super-strong Wilf classes in S_n with order a given power of 2. As a by-product, we give a recursively defined set of representatives of super-strong Wilf equivalence classes. Finally we establish the connection of super-strong Wilf equivalence with the geometric notion of *shift equivalence* for permutations.

Our anticipated results could lead to answers or new approaches to existing conjectures in the area of patterns in permutations and words, such as the *Rearrangement Conjecture*, stating that two words are Wilf-equivalent if and only if they are rearrangements of each other and the conjecture that Wilf and strong Wilf equivalence coincide.

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