A review of recent advances of Euler's polygonal approximations

Sotirios Sabanis (Σωτήριος Σαμπάνης)
National Technical University of Athens/University of Edinburgh/ Alan Turing Institute, s.sabanis@math.ntua.gr

The idea of using a new form of Euler's polygonal approximations, one which allows coefficients to depend directly on the step size, was highlighted in [1] and [2]. It was used to simplify proofs and extend results on existence of SDEs with monotone coefficients. Since then, it has been applied in several other directions, which include solvability questions for stochastic delay differential equations (SDDEs), see [3], new explicit numerical schemes for SDEs with superlinear coefficients, see e.g. [4], [5] and references therein, as well as in the construction of MCMC algorithms, see e.g. [6] and [7]. More recently, this new form of Euler's polygonal approximations has been used to create new, stochastic (adaptive) optimization algorithms with superior performance, in many cases, than other leading optimization algorithms within the context of fine tuning of artificial neural networks, see [8] and [9]. We will review some key developments of this new methodology, in particular with regards to data science and ML.

[1] N. V. Krylov. *Extremal properties of the solutions of stochastic equations. Theory of Probability and its Applications.* 29(2):205–217, 1985.

[2] N. V. Krylov. *A simple proof of the existence of a solution to the Itô's equation with monotone coefficients. Theory of Probability and its Applications.* 35(3):583–587, 1990.

[3] I. Gyöngy and S. Sabanis. *A Note on Euler Approximations for Stochastic Differential Equations with Delay. Appl. Math. Optim.* 68, 391–412, 2013.

[4] S. Sabanis. *Euler approximations with varying coefficients: the case of superlinearly growing diffusion coefficients. Annals of Applied Probability.* 26(4), 2083–2105, 2016.

[5] S. Sabanis and Y. Zhang. *On explicit order 1.5 approximations with varying coefficients: the case of super-linear diffusion coefficients. Journal of Complexity.* 50, 84–115, 2019.

[6] N. Brosse, A. Durmus, É. Moulines, and S. Sabanis. *The tamed unadjusted Langevin algorithm. Stochastic Processes and their Applications.* 129(10):3638–3663, 2019.

[7] S. Sabanis and Y. Zhang. *Higher order Langevin Monte Carlo algorithm. Electronic Journal of Statistics.* 13(2):3805–3850, 2019.

[8] A. Lovas, I. Lytas, M. Rasonyi, and S. Sabanis. *Taming neural networks with tusla: Non-convex learning via adaptive stochastic gradient langevin algorithms. arXiv preprint arXiv:2006.14514*, 2020.

[9] D.-Y. Lim and S. Sabanis *Polygonal Unadjusted Langevin Algorithms: Creating stable and efficient adaptive algorithms for neural networks. arXiv preprint arXiv: arXiv:2105.13937*, 2021.