

Abstract: In this talk we discuss how it came about, starting with John Horton Conway's skein theory of the Alexander polynomial, that

new invariants of knots arose (the Jones polynomial among them) that had skein relations but were related to statistical mechanics and then

(through the work of Witten) to gauge theory and quantum field theory. This relationship of knot theory and physical theory is intimately tied with

a mystery discovered by Herman Weyl in the early part of the 20th century. Weyl discovered that if one takes a line element in spacetime of the form

$A = Pdx + Qdy + Rdz - Sdt$ and writes down dA in the sense of the differential forms of Grassmann, then dA expresses the mathematical form of the Electromagnetic Field.

This means that the field is expressed by the holonomy of the form A around loops in spacetime. Weyl was so impressed with his observation that he suggested building

a Geometry that would unify his line element A and the metric of General Relativity to make a unified field theory. But why should spacetime lengths change under transport, asked Einstein,

and the Weyl theory did not quite succeed. Yet it did succeed by quantum reformulation where the key was to see $\exp(i \text{ Holonomy Integral on a Path}(A))$. With the use of $i^2 = -1$, the

holonomy is transformed into the phase of a wave function in quantum mechanics. Experimental confirmation of the influence of a gauge potential A on quantum interference came much

later with the Aharonov-Bohm effect. Theoretical influence of this idea came with the generalization of A to a Lie algebra valued 1-form and the corresponding generalized gauge theories

such as Yang-Mills theory. Then the physical field is not dA but $dA + A \wedge A$ and the holonomy remains important. Witten suggested the use of a spatial gauge A and that measuring its holonomy along a knot K , $W_{\{K\}}(A)$, should produce invariants such as the Jones polynomial. But $W_{\{K\}}(A)$ is a mixed quantity, depending on the gauge connection A and changes in relation to the curvature of that connection. Witten

understood that a formal answer was to integrate appropriately over all the connections, forming an integral

$Z_{\{K\}} = \text{Integral}(D A \exp(k CS(A)) W_{\{K\}}(A))$ where $CS(A)$ is the integral of the Chern-Simons form associated with A . This integral of Witten is a functional integral in the quantum field theory associated with A . It has deep formal properties that inform indeed not only the Jones polynomial, but a host of other invariants as well, and the seeds of relationships with the three manifold invariants of Reshetikhin and Turaev, and the so-called Vassiliev invariants of knots and links. Topological Quantum Field Theory was born. This is the story of a revolution in knot theory that started with Conway in 1969 and began again with Witten in 1988. This talk will discuss these matters up to around 1990 and will mention some more recent developments such as Khovanov homology whose physical interpretations are not yet fully articulated. The talk will be self-contained against a background of basic abstract algebra and advanced calculus.