We will tour some topics of recent research in elementary number theory. For example:

Binomial coefficients have very many arithmetic properties. Given a sequence of nonzero integers $A = (a_n)_{n \geq 0}$ we associate generalized binomials

$$\binom{m+n}{n}_A := \frac{a_{m+n}a_{m+n-1} \cdots a_{m+1}}{a_na_{n-1} \cdots a_1}.$$ 

Are some properties of ordinary binomials preserved for special sequences $A$?

In 1862, Wolstenholme proved that for all primes $p \geq 5$

$$\binom{2p-1}{p-1} \equiv 1 \pmod{p^3}.$$ 

We will look at similar congruences when using fundamental Lucas sequences $U = U(P, Q)$ ($U_0 = 0, U_1 = 1, U_{n+2} = PU_{n+1} - QU_n, P, Q$ integers) and generalized binomials with respect to $U$. 