Topics in Elementary Number Theory.

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We will tour some topics of recent research in elementary number theory. For example:

Binomial coefficients have very many arithmetic properties. Given a sequence of nonzero integers $A = (a_n)_{n\geq 0}$ we associate generalized binomials

$$\binom{m+n}{n}_A := \frac{a_{m+n}a_{m+n-1}\dots a_{m+1}}{a_n a_{n-1}\dots a_1}.$$

Are some properties of ordinary binomials preserved for special sequences A?

In 1862, Wolstenholme proved that for all primes $p \geq 5$

$$\binom{2p-1}{p-1} \equiv 1 \pmod{p^3}.$$

We will look at similar congruences when using fundamental Lucas sequences $U = U(P,Q)$ ($U_0 = 0$, $U_1 = 1$, $U_{n+2} = PU_{n+1} - QU_n$, P, Q integers) and generalized binomials with respect to U.