Rarefaction wave interaction for the unsteady transonic small disturbance equations

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Abstract: We study a Riemann problem for the unsteady transonic small disturbance equations that results in a diverging rarefaction problem. The self-similar reduction leads to a boundary value problem with equations that change type (hyperbolic-elliptic) and a sonic line that is a free boundary. We summarize the principal ideas and present the main features of the problem. The flow in the hyperbolic part can be described as a solution of a degenerate Goursat boundary problem, the interaction of the rarefaction wave with the subsonic region is illustrated and the subsonic flow is shown to satisfy a second order degenerate elliptic boundary problem with mixed boundary conditions.

Key–Words: two-dimensional Riemann problem, rarefaction waves, unsteady transonic small disturbance equations, mixed-type equations, degenerate Goursat boundary problem, free boundary.

1 Introduction

We study a diverging rarefaction problem for a special system of conservation laws, the unsteady transonic small disturbance (UTSD) equations:

$$u_t + uu_x + v_y = 0\,, \qquad (1)$$
$$-v_x + u_y = 0\,, \qquad (2)$$

with appropriate Riemann data. Here, $(x, y) \in \mathbb{R}^2$, $t \geq 0$ and u and v denote the components of the physical velocity, which are functions of (t, x, y).

Systems of conservation laws in one spatial variable have been studied extensively and the theory regarding the existence of solutions is well developed. For systems in more than one spatial variable, very little is known and there is a great interest to understand the complicated behavior observed in phenomena represented in multi-dimensional conservation laws for their mathematical, but also for their physical importance. It is well known that Riemann problems serve as building blocks in solutions to general conservation laws with arbitrary data in one-space dimension. Therefore, such problems could indicate what types of singularities arise in multi-dimensional general systems. Exposition of the current state of the theory can be found in the books [12, 23].

There is an intensive program by many members of the community in conservation laws to investigate this area of research. As a first step, the focus is on Riemann problems of simplified models with the incentive to move from the particular to the general. The long-term goal is to understand Riemann solutions of prototype problems, and then turn to a difficult task of gluing together the various discontinuity types to describe solutions of general systems with arbitrary data. This approach was also employed for the case of one space dimension having shocks and rarefactions.

Several groups in conservation laws (Čanić–Keyfitz–Kim [1]–[3], G.-Q. Chen–Feldman [5]–[9], S. Chen [10]–[11], Elling–T. P. Liu [14], Zheng et al [25]–[28], etc.) have adapted the self-similar approach to study shock reflection and rarefaction problems for some simplified models of Euler equations in recent years. In self-similar coordinates, the equations reduce to mixed hyperbolic-elliptic type across the sonic line. The formation of a shock as a free boundary in shock reflection problems raises the need to investigate free boundary problems with mixed boundary conditions and use compactness methods to construct a convergent subsequence that induces a solution. In G.-Q. Chen and Feldman [9], global existence and stability to regular shock reflection are established for potential flow. Results about local solutions for shock reflection for other models can be found in [1]–[3], [17]-[20]. Rarefaction problems have been studied mostly by Zheng and his group [25]–[28] and no global existence result is known yet.

In this article, we study the UTSD equations (1)–(2) and we choose special Riemann data so that the waves are outward-traveling rarefactions. It should be noted that the UTSD equations can be obtained by an asymptotic reduction of the compressible gas dynamics equations when having weak shocks and small deviation from one-dimensional flow [6, 7] and therefore, serve as a prototype model. For more details, we refer to Morawetz [22] and Hunter–Tesdall [16].

We remark that this work is in progress and we present the main ideas and features of this diverging rarefaction problem that we obtained so far. The aim is to give an emphasis on the new features that arise when dealing with rarefactions, in contrast to shock reflection phenomena and the different techniques that need to be implemented to solve the free boundary problem that arises. We first write the problem in self-similar coordinates and then, establish the solution in part of the hyperbolic region, far from the origin (Section 2). Next, we show that the equations are of mixed-type in the interaction region. In contrast to shock reflection problems, the free boundary is the sonic line along which the problem is degenerate. The flow in the hyperbolic part can be described as a solution of a degenerate Goursat boundary problem with characteristic data on a curve and matching condition on the sonic boundary (Section 3.1). Furthermore, we describe how the rarefaction interacts with the subsonic region and we choose appropriate mixed boundary conditions (Section 3.2). Finally, we propose an iteration scheme to solve the free boundary problem (Section 3.3). We mention that new techniques need to be developed to establish existence of a solution of this rarefaction problem and, at this stage, we present the results established so far. We expect though that the main features observed here will be useful in subsequent attempts to tackle more general rarefaction problems.

It is expected that a weak shock will form at or near the sonic line somewhere in the interaction region. This has been observed numerically by Tesdall [24]. In this project, we focus only on the local solution and we do not investigate the formation of this shock.

2 The UTSD Equations

If the Riemann problem for the UTSD equations (1)–(2) has the following initial data:

$$U|_{t=0} = \begin{cases} (-1,-a), & \text{for} \quad 0 < y < -x/a; \\ (-1,a), & \text{for} \quad x/a < y < 0; \\ (0,0), & \text{otherwise}, \end{cases}$$

with $a > 0$, two rarefaction waves will be created near the lines of initial discontinuities $y = \pm\frac{x}{a}$. As time increases, the two rarefaction waves will spread out and interact near the origin.

There is a self-similar structure for the expected solution. Hence, we study the UTSD equations in self-similar coordinates $(\xi, \eta) \equiv (x/t, y/t)$:

$$\begin{aligned} (u-\xi)u_\xi - \eta u_\eta + v_\eta &= 0, & (3)\\ -v_\xi + u_\eta &= 0. & (4) \end{aligned}$$

We rewrite the above equations in the matrix form

$$U_\xi + A(U,\xi,\eta)\, U_\eta = 0, \tag{5}$$

where

$$A(U,\xi,\eta) = \frac{1}{\xi - u}\begin{bmatrix} \eta & -1 \\ u-\xi & 0 \end{bmatrix},$$

and we compute the eigenvalues of A:

$$\lambda_{1,2} = \frac{\eta \pm \sqrt{\eta^2 + 4\xi - 4u}}{2(\xi - u)}. \tag{6}$$

It is clear that, when linearized about a constant state, system (3)–(4) changes type across the sonic parabola $u = \xi + \eta^2/4$, and is hyperbolic if and only if $u < \xi + \eta^2/4$.

Because the problem is symmetric, we only need to study the solution in the upper half plane with the rarefaction wave from above. Since the rarefaction wave is parallel to the line $\eta = -\frac{\xi}{a}$ far from the origin, we assume the dependence $U = U(\xi + a\eta)$ in order to find the rarefaction wave away from the interaction. Using the expression $U(\xi + a\eta)$ to solve equations (3)–(4), we obtain that the rarefaction wave is given by

$$u_0 = \xi + a\eta - a^2 \quad \text{and} \quad v_0 = a u_0, \tag{7}$$

for $a^2 - 1 < \xi + a\eta < a^2$. The right border of the rarefaction wave is $\xi + a\eta = a^2$ along which $(u_0, v_0) = (0, 0)$.

When the rarefaction wave stretches down to ξ-axis, an interaction with the rarefaction wave in the lower half plane occurs, giving rise to a very complicated structure. An elliptic region appears near the origin due to this interaction. The right border of the elliptic region is determined in the following way. On the border, we assume that the system is elliptic degenerate and the data matches the right state $u = 0$. This gives a sonic parabola $\xi + \eta^2/4 = 0$, which intersects the right boundary of the rarefaction wave $\xi + a\eta = a^2$ at the point $A(-a^2, 2a)$ (see Figure 1). It turns out that the information at the point A travels into the hyperbolic region and curves the rarefaction

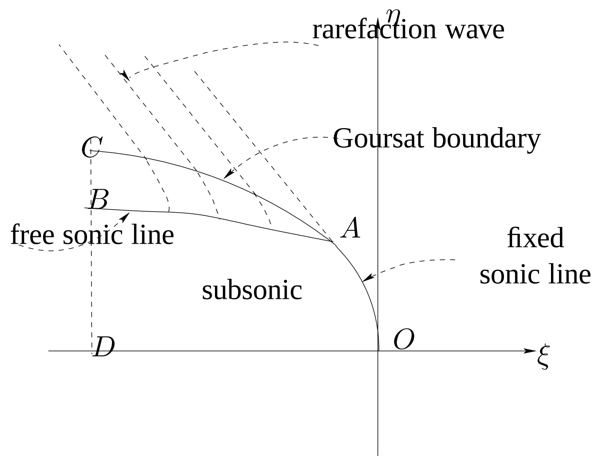

Figure 1: Domain of the rarefaction wave interaction in ξ-η plane

wave. This phenomenon is due to the fact that the flow is transversal to the family of parabolas $\xi + \eta^2/4 = k$ and travels inward.

It is convenient to consider the problem in parabolic coordinates (ρ, η), where $\rho = \xi + \eta^2/4$. Then the flow is in the negative ρ direction. If we consider the nonphysical flow direction from the far field above to the origin, we cannot obtain a continuous transonic flow for this problem. This can be seen from the following argument. We rewrite equations (3)–(4) in (ρ, η)-coordinates

$$(u-\rho)u_\rho - \frac{\eta}{2}u_\eta + v_\eta = 0 \quad (8)$$

$$\frac{\eta}{2}u_\rho - v_\rho + u_\eta = 0. \quad (9)$$

Assume that the flow travels along the negative η direction. Therefore, the rarefaction wave will keep straight level curves until it hits the sonic line, which turns out to be the horizontal line given by $L_0 : \eta = 2a$. Equations (8)–(9) can be reduced to a second order equation for a potential function φ:

$$(\varphi_\rho - \rho)\varphi_{\rho\rho} + \varphi_{\eta\eta} + \frac{1}{2}\varphi_\rho = 0, \quad (10)$$

where $\varphi_\rho = u$ and $\varphi_\eta = v - \frac{\eta}{2}u$. We denote the corresponding potential above the sonic line L_0 by

$$\varphi_0 = \frac{1}{32}(4\rho - (\eta - 2a)^2)^2,$$

and we remark that equation (10) is elliptic below the line L_0. It is easy to check that

$$\psi = \frac{1}{2}\rho^2(1 + \varepsilon(\eta - 2a))$$

is a super-solution, when boundary condition on L_0 is $\varphi = \frac{1}{2}\rho^2$. Hence, we have $\varphi \le \psi$, which implies $\varphi_\eta|_{L_0} \ge \frac{1}{2}\varepsilon\rho^2 > 0$. On the other hand, $(\varphi_0)_\eta|_{L_0} = 0$, which means that v cannot be continuous across L_0. Hence, the physical flow should travel along negative ρ direction instead of negative η direction.

Now, we investigate the region affected by the point A. To determine this region, we need to find the characteristic lines starting from A. Given $u = \xi + a\eta - a^2$ and using (6), we obtain the eigenvalues

$$\lambda_1 = -\frac{1}{a} \quad \text{and} \quad \lambda_2 = -\frac{1}{\eta - a}.$$

Therefore, at the point A, the characteristic of the 1-family is just the right border of the rarefaction wave and the characteristic of the 2-family is the arc of the parabola

$$\widehat{AC} : \xi + \frac{(\eta - a)^2}{2} = -\frac{a^2}{2}.$$

The straight rarefaction wave will be curved below $\widehat{AC}$ due to the effect of the information at point A and become hyperbolic degenerate when it reaches the sonic line $\widehat{AB}$, which is a free boundary.

Numerical simulations by Tesdall [24] indicate the complicated structure of the problem and appearance of a weak shock near or at the free boundary on the left side. However, in this paper we consider the problem only near the origin and for that reason we impose a cutoff boundary (line $\widehat{CD}$).

In view of the above analysis, we formulate the problem as follows.

Problem: Find a Lipschitz continuous solution (u, v) of system (3)–(4) such that

$$\begin{aligned} &(u,v)|_{\widehat{AC}} = (u_0, v_0), \\ &(u,v)|_{\widehat{OA}} = (0,0), \\ &v|_{\widehat{OD}} = 0, \\ &u_\eta|_{\widehat{OD}} = 0, \end{aligned}$$

with some appropriate conditions along the cutoff boundary $\widehat{BD}$.

3 Strategy and Preliminary Ideas

In this section, we present our main ideas regarding the proof of existence of solutions to the above free boundary problem. The study can be divided into several problems that need to be resolved and pieced together in order to establish the solution in the interaction region. First, we fix the sonic line $\widehat{AB}$ and we solve the fixed boundary problem. Solving the fixed boundary problem consists of two steps as the domain is divided in two parts: degenerate hyperbolic and degenerate elliptic problems in domains described by ABC and $ABDO$, respectively. Then we employ an iteration scheme to establish the sonic line as a free boundary.

3.1 Degenerate Hyperbolic Problem

We consider the degenerate hyperbolic problem in the domain ABC. Recall that $\widehat{AC}$ is a Goursat boundary with data given by

$$(u, v)|_{\widehat{AC}} = (u_0, v_0),$$

$\widehat{AB}$ is a fixed sonic line, which is degenerate hyperbolic, with condition

$$u|_{\widehat{AB}} = \xi + \frac{\eta^2}{4},$$

and $\widehat{BC}$ is the cutoff boundary along which no conditions should be imposed. The problem is genuinely difficult because of the degeneracy on the sonic line, and, in particular, controlling the hyperbolicity, i.e., the sign of $\xi + \frac{\eta^2}{4} - u$, becomes complicated.

We rewrite equations (8)–(9) as

$$U_\rho + B(U, \rho, \eta)\, U_\eta = 0,$$

where the matrix B is given by

$$B(U, \rho, \eta) = \frac{1}{u - \rho} \begin{bmatrix} -\eta/2 & 1 \\ -\eta^2/4 + \rho - u & \eta/2 \end{bmatrix}.$$

Then, we find the eigenvalues of B

$$\lambda_1 = -\frac{1}{\sqrt{\rho - u}} \quad \text{and} \quad \lambda_2 = \frac{1}{\sqrt{\rho - u}},$$

the right eigenvectors

$$r_1 = \begin{bmatrix} 1 \\ \frac{\eta}{2} + \sqrt{\rho - u} \end{bmatrix},\ r_2 = \begin{bmatrix} 1 \\ \frac{\eta}{2} - \sqrt{\rho - u} \end{bmatrix},$$

and the left eigenvectors

$$l_1 = [-\frac{\eta}{2} + \sqrt{\rho - u} \quad 1]\,, \quad l_2 = [-\frac{\eta}{2} - \sqrt{\rho - u} \quad 1].$$

It should be noted that the system is hyperbolic if and only if $u < \rho$. Moreover, we compute the Riemann invariants

$$R_1(U, \rho, \eta) = -\frac{\eta}{2} u - \frac{2}{3}(\rho - u)^{3/2} + v\ ,$$

$$R_2(U, \rho, \eta) = -\frac{\eta}{2} u + \frac{2}{3}(\rho - u)^{3/2} + v,$$

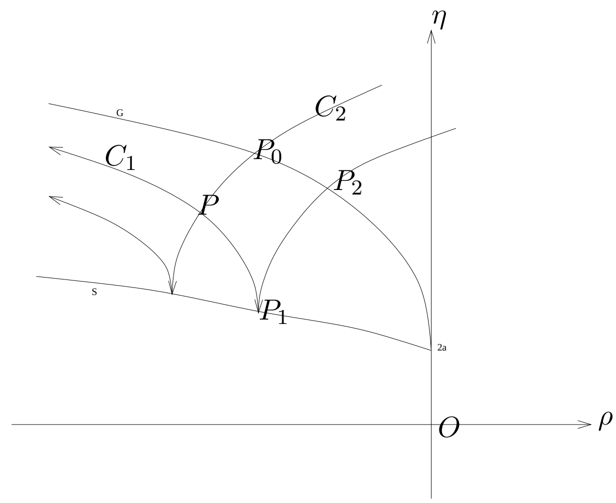

Figure 2: The two families of characteristic curves C_1, C_2 in the hyperbolic domain ABC in ρ-η plane.

and show that they satisfy the evolution equations

$$(R_1)_\rho + \lambda_1 (R_1)_\eta + \frac{\rho - \frac{3}{2}u}{\sqrt{\rho - u}} = 0\ ,$$

$$(R_2)_\rho + \lambda_2 (R_2)_\eta - \frac{\rho - \frac{3}{2}u}{\sqrt{\rho - u}} = 0.$$

Also, we establish the relations

$$D_U R_1 r_1 = l_1 r_1 = 2\sqrt{\rho - u}$$

and

$$D_U R_2 r_2 = l_2 r_2 = -2\sqrt{\rho - u},$$

indicating certain monotonicity of Riemann invariants along the two families of characteristics (see Figure 2). Using this information, we were expecting to find R_1 and R_2 inside the domain starting from the initial condition along the boundary $\widehat{AC}$ and using the boundary condition $R_1 = R_2$ along $\widehat{AB}$ (see Figure 1). However, since R_1 is increasing along the 1-family of characteristics, R_2 is decreasing along the 2-family of characteristics and $R_1 = R_2$ along $\widehat{AB}$, more analysis is required to control the sign of $R_1 - R_2$ and therefore, of $\rho - u$.

Hence, we follow a different approach to control the sign of $\rho - u$. We actually use arguments similar to those in [28] by Y. Zheng and [13] by Z. Dai and T. Zhang. More precisely, we denote by

$$D_+^u = \sqrt{\rho - u}\,\partial_\rho + \partial_\eta,\ D_-^u = -\sqrt{\rho - u}\,\partial_\rho + \partial_\eta.$$

By virtue of equations (8)–(9), we derive

$$D_-^u(D_+^u u) = -\frac{1}{\sqrt{\rho - u}} u_\rho D_+^u u, \qquad (11)$$

$$D_+^u(D_-^u u) = \frac{1}{\sqrt{\rho - u}} u_\rho D_-^u u. \qquad (12)$$

Given an interior point P, the 2-characteristic C_2 will intersect the Goursat boundary $\widehat{AC}$ at P_0 and equation (12) yields

$$D_-^u u(P) = D_-^u u(P_0) e^{\int_{C_2} \frac{u_\rho}{\sqrt{\rho - u}} ds}.$$

But we have $D_-^u u = -2\sqrt{-\rho} < 0$ on the Goursat boundary, implying $D_-^u u(P) < 0$. Similarly, by equation (11), we get that $D_+^u u(P)$ has the same sign along 1-family of characteristics C_1 until we reach the point P_1 on the sonic line $\widehat{AB}$. Since $D_+^u u(P_1) = D_-^u u(P_1)$ on the sonic line, tracing along another 2-family of characteristics, we get $D_+^u u(P) < 0$. It follows immediately that

$$u_\eta(P) = \frac{1}{2}(D_+^u u(P) + D_-^u u(P)) < 0 \,,$$

which implies the result $u(P) < \rho$.

3.2 Degenerate Elliptic Problem

After constructing a solution of the degenerate hyperbolic problem, we consider the degenerate elliptic problem in the domain $ABDO$. As it is usual in the study of two-dimensional systems of conservation laws, we derive a second order equation for one of the variables and impose mixed boundary conditions along the boundary. The theory of second order elliptic equations with mixed boundary conditions by Gilbarg, Trudinger and Lieberman is used to establish existence of a solution (see [15]).

More precisely, by differentiating equation (3) with respect to ξ and (4) with respect to η, we obtain a second order equation for u:

$$((u-\xi)u_\xi)_\xi - \eta u_{\xi\eta} + u_{\eta\eta} = 0. \qquad (13)$$

It is easy to check that (13) is elliptic if and only if $\xi + \frac{\eta^2}{4} < u$. We prescribe the following conditions along the boundary $\widehat{ABDO}$. Along the arch $\widehat{AB}$, we prescribe a Dirichlet condition $u = \xi + \frac{\eta^2}{4}$, which states that the boundary $\widehat{AB}$ is elliptic degenerate. Along the cutoff boundary $\widehat{BD}$, we impose a Dirichlet condition $u = f(\eta)$, for an appropriate function f. We require continuity along the parabolic arch $\widehat{OA}$, implying the Dirichlet condition $u = 0$. Finally, we impose a symmetry condition $u_\eta = 0$ along $\widehat{OD}$.

To find a solution of the second order equation (13) in the domain $ABDO$, we first need to modify the coefficients by cutoff functions to enforce ellipticity. Noticing that $\phi(\xi,\eta) = \xi + \frac{\eta^2}{4}$ is a sub-solution of (13), a comparison principle enables the estimate

$$u > \xi + \frac{\eta^2}{4}.$$

This guarantees the ellipticity of (13) and removal of the cutoff functions.

Along the sonic boundary $\widehat{OAB}$, the nonlinear structure of (13) is crucial to derive the necessary estimates (see G.-Q. Chen and Feldman [9]). However, instead of one side degeneracy as in [9], we have elliptic degeneracy on both sides of the boundary. We believe that some arguments in [21] by Kohn and Nirenberg will help resolve this issue.

Once we obtain the solution u of the above second order equation with mixed boundary conditions, we find v by integrating equation (3), i.e.,

$$v(\xi,\eta) = \int_0^\eta (y u_y - (u-\xi)u_\xi) dy.$$

3.3 Iteration Scheme

Once we can solve both the hyperbolic and elliptic problems, we need an iteration scheme to update the position of the sonic boundary. This will lead to a mapping for which we need to show that it has a fixed point.

An interesting question is how to update the sonic boundary. We recall that in the shock reflection problems (see Čanić, Keyfitz, Kim, Lieberman and Jegdić [1]-[4], [17]-[19]), the free boundary is the reflected shock and the standard way to update the free boundary is to use the Rankine-Hugoniot jump conditions and the solution of the fixed boundary problem. Here, the situation is different and we propose that the way to update the position of the free boundary is to match the value of v on both sides. Since this matching is along the degenerate boundary, one needs to know additional information about the values of v from both sides of the sonic line.

4 Conclusion

In this paper, we consider a Riemann problem for the UTSD equations that leads to interacting rarefaction waves. We rewrite the problem in self-similar coordinates and obtain a mixed-type system and a free boundary problem. The free boundary problem consists of two parts – degenerate hyperbolic and degenerate elliptic. Our preliminary analysis shows that both problems are genuinely difficult and require development of novel techniques in proving existence of a solution.

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