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LETTER TO THE EDITOR

A REMARK ON CONSUL'S (1990) COUNTER-EXAMPLE

Dear Sir,

Kosambi (1949) and Patil (1962) have proved that the equality of mean and variance for a power-series distribution (family) characterizes the Poisson distribution. Recently, Consul (1990) has attempted to disprove this result via a counter-example. The purpose of this letter is to point out that Consul's counter-example is misleading and does not contradict the result in question. Also, there appear to be some other blemishes, possibly due to printing errors, associated with Consul's example; as, it stands, his values of $P(X = x)$ do not produce a probability distribution.

Consul seems to be overlooking a crucial assumption in the Kosambi-Patil result. Even if it is not explicitly stated by these authors, it is clear that in their result there is a requirement that the relation be valid for all parameter values lying in an open interval. If the condition is not interpreted this way, then it does not make any sense in assuming that the distribution is power-series; indeed, any arbitrary discrete distribution on $\{0, 1, 2, ...\}$ could be expressed as $\left\{\frac{a_j\theta_0^j}{A(\theta_0)}\right\}$ (in obvious notation) for a particular value of θ_0 and hence it would not be correct to be referred to as a power-series distribution. To avoid future confusions and controversies, we may restate the Kosambi-Patil result in a more definitive form as follows:

Theorem. Let $\{X_\theta : \theta \in (a,b), 0 \leq a < b\}$ be a family of a non-negative integer-valued random variables such that (for each θ)

$$P_\theta(X_\theta = n) = \frac{a_n \theta^n}{A(\theta)}, \quad n = 0, 1, ...,$$

where $a_n \geq 0$ and independent of θ for every $n \geq 0$. Then

$$E_\theta(X_\theta) = V_\theta(X_\theta) \quad \text{(for all } \theta)$$

if and only if (each) X_θ has a Poisson distribution.

An error of the type in Consul's example has also appeared in an example given earlier by Binet (1974). This latter author has now known about the error and has produced a correction note in Binet (1978) to state this.

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