Shape sensitivity analysis of the Hardy constant

Gerassimos Barbatis

Department of Mathematics, University of Athens
gbarbatis@math.uoa.gr

Abstract

We consider the Hardy constant associated with a domain in the n-dimensional Euclidean space and we study its variation upon perturbation of the domain. We prove a Frechet differentiability result and establish a Hadamard-type formula for the corresponding derivatives. We also prove a stability result for the minimizers of the Hardy quotient. Finally, we prove stability estimates in terms of the Lebesgue measure of the symmetric difference of domains. This is a joint work with P.D. Lamberti (University of Padova).

Wednesday 5pm-6pm

Some non local Monge Ampere type equations

Luis Caffarelli

Department of Mathematics, University of Texas at Austin
caffarel@math.utexas.edu

Abstract

In the spirit of optimal control, we will discuss two type of non local elliptic equations with affine invariance, i.e., the equation consists of an infimum over a family of elliptic operators that is invariant under transformations of determinant one.

Tuesday 9:30am-10:30am

The spectrum of the Laplacian on noncompact manifolds

Nelia Charalambous

Department of Mathematics and Statistics, University of Cyprus
charalambous.nelia@ucy.ac.cy

Abstract

The essential spectrum of the Laplacian on functions has been extensively studied. It is known that on hyperbolic space a spectral gap appears, whereas it has been conjectured that on manifolds with uniformly subexponential volume growth and Ricci curvature bounded below the essential spectrum is the nonnegative real line.

In our work with Zhiqin Lu we prove a generalization of Weyl's criterion for the essential spectrum. We then apply this generalized criterion to expand the set of manifolds on which the essential spectrum is the nonnegative real line. We also use our criterion to compute the essential spectrum of complete shrinking Ricci solitons and weighted manifolds, as well as to study the essential spectrum of the Laplacian on forms.

Wednesday 6pm-7pm

The Isometric Embedding Problem in $\mathbb{R}^3$ of two-dimensional Riemannian manifolds with slowly decaying negative Gauss curvature.

Cleopatra Christoforou

Department of Mathematics and Statistics, University of Cyprus
christoforou.cleopatra@ucy.ac.cy

Abstract

The isometric embedding problem is a fundamental problem in Differential Geometry, that has been extensively studied. I will describe this problem for complete two-dimensional Riemannian manifolds with negative Gauss curvature in the setting of hyperbolic systems of balance laws. First, I will give an overview of the existing results regarding global existence and non-existence issues depending on the decay rate of the Gauss curvature. Together with Marshall Slemrod, we prove global existence of isometric immersions in $\mathbb{R}^3$ for complete two-dimensional Riemannian manifolds with negative Gauss curvature, which decays at a slow rate and this result provides an improvement on the previously known decay rates. **Wednesday 6pm-7pm**

New interpretations of the infinity Laplacian PDE

Lawrence C. Evans

Department of Mathematics, University of California, Berkeley
evans@math.berkeley.edu

Abstract

I will discuss several old and new interpretations of the infinity Laplacian, including the geometric observation of Drucker–Williams that gradient flows give asymptotic curves on the graphs of smooth solutions. **Monday 9:30am-10:30am**

Trace Hardy Sobolev Mazy'a inequalities for fractional Laplacians

Stathis Filippas

Department of Mathematics and Applied Mathematics, University of Crete
filippas@tem.uoc.gr

Abstract

We establish trace Hardy and trace Hardy-Sobolev-Maz'ya inequalities with best Hardy constants, for domains satisfying suitable geometric assumptions such as mean convexity or convexity. We then use them to produce fractional Hardy-Sobolev-Maz'ya inequalities with best Hardy constants for various fractional Laplacians. In the case where the domain is the half space our results cover the full range of the exponent $s \in (0, 1)$ of the fractional Laplacians. **Wednesday 12-1pm**

Weighted norms and decay properties for solutions of the Boltzmann equation

Irene Gamba
Department of Mathematics, University of Texas at Austin
gamba@math.utexas.edu

Abstract

We will discuss recent results regarding generation and propagation of summability of moments to solution of the Boltzmann equation for variable hard potentials. These estimates are in direct connection to the understanding of high energy tails and decay rates to equilibrium. **Monday 10:30am-11:30am**

Rigidity results for conformal immersions

Tobias Lamm
Department of Mathematics, Karlsruhe Institute of Technology
tobias.lamm@kit.edu

Abstract

In my talk I will describe recent rigidity results for certain two-dimensional surfaces in euclidean spaces. These results were obtained in collaborations with Huy Nguyen and Reiner Schätzle. I will try to emphasize the importance of the Hardy space estimate of Müller and Sverak for these kind of problems. **Monday 3:30pm-4:30pm**

Mumford-Shah minimizers in dimension 3: recent results and open questions

Antoine Lemenant
Laboratoire Jacques-Louis Lions, Université Paris Diderot-Paris 7
lemenant@ljll.univ-paris-diderot.fr

Abstract

In this talk I will present briefly the Mumford-Shah functional and focus on regularity issues for the minimizers. Then I will present a new result (2014) about global minimizers (i.e. blow-up limits of minimizers) whose singular set is contained in a cone. We will show in particular the following rigidity result: if the singular set of a global minimizer is contained in a half-plane, then it must be the half-plane itself. One of the key ingredient in the proof is a monotonicity formula of Alt-Caffarelli-Friedman type adapted for the Neumann problem, and which is valid in the special case when the "free boundary" is contained in a (N-1)-rectifiable cone. **Wednesday 3:30pm-4:30pm**

Improved convergence theorem for perimeter minimizing clusters

Francesco Maggi
Department of Mathematics, University of Texas at Austin
maggi@math.utexas.edu

Abstract

A well-known fact about sequences of perimeter almost-minimizing sets is that L1 convergence (convergence to zero of the volume of the symmetric difference) improves to C1 convergence (existence of boundary diffeomorphisms which converge to the identity map in C1) whenever the limit set has smooth boundary. This is a classical application of the small excess regularity criterion, which is useful in showing the equivalence of L1-local and C1-local minimality conditions, as well as in proving quantitative stability inequalities, and in providing qualitative descriptions of minimizers in surface tension driven problems. The smoothness assumption on the limit set is automatically valid in dimension less or equal than 7, but may fail otherwise. Our understanding of these singularities is to lacunary to allow for an extension of the above "improved convergence theorem" when the limit set is singular. When one moves from the framework of sets to that of clusters (modeling, say, soap bubble compounds) singularities appear even in dimension 2. However, in the case of clusters, we have a very good understanding of singularities in dimension 2 and 3, based on Jean Taylor's theorem on the validity of Plateau's laws. Starting from this sharp local description of bubble clusters we explain how to prove improved convergence theorems for sequences of singular sets of perimeter almost-minimizing bubble clusters in R2 and R3, and briefly discuss some of the possible applications of these results.

Wednesday 10:30am-11:30am

Peter Markowich
Department of Applied Mathematics and Theoretical Physics,
University of Cambridge
P.A.Markowich@damtp.cam.ac.uk

Abstract

Tuesday 10:30am-11:30am

Optimal L^p Hardy-type inequalities

Yehuda Pinchover
Department of Mathematics, Technion - Israel Institute of Technology
pincho@tx.technion.ac.il

Abstract

Let Ω be a domain in $\mathbb{R}^n$ or a noncompact Riemannian manifold of dimension $n \geq 2$, and $1 < p < \infty$. Consider the functional

$$\mathcal{Q}(\varphi) := \int_\Omega \left(|\nabla \varphi|^p + V(x)|\varphi|^p \right) \mathrm{d}\nu$$

defined on $C_0^\infty(\Omega)$, and assume that $\mathcal{Q} \geq 0$. In this talk we discuss generalizations of some of the results obtained in [1] for the linear case ($p = 2$) to the quasilinear case ($p \neq 2$). In particular, we obtain "as large as possible" nonnegative (optimal) Hardy-type weight W satisfying

$$\mathcal{Q}(\varphi) \geq \int_\Omega W|\varphi|^p \,\mathrm{d}\nu \qquad \forall \varphi \in C_0^\infty(\Omega).$$

Our main results deal with the case where $V = 0$, and Ω is a general punctured domain (for $V \neq 0$ we obtain only some partial results). In the case $1 < p \leq n$, an optimal Hardy-weight is given by

$$W := \left(\frac{p-1}{p} \right)^p \left| \frac{\nabla G}{G} \right|^p,$$

where G is the associated positive minimal Green function with a pole at 0. On the other hand, for $p > n$, several cases should be considered, depending on the behavior of G at infinity in Ω. The results are extended to annular and exterior domains.

This is a joint work with Baptiste Devyver.

References

[1] B. Devyver, M. Fraas, Y. Pinchover, *Optimal Hardy Weight for Second-Order Elliptic Operator: an answer to a problem of Agmon*, J. Functional Analysis **266** (2014), 4422–4489.

[2] B. Devyver, Y. Pinchover, *Optimal L^p Hardy-type inequalities*, 32 pp., arXiv: 1312.6235.

Tuesday 12-1pm

Martingale Optimal Transport and Robust Hedging

Halil Mete Soner

Department of Mathematics, ETH-Swiss Institute of Technology, Zurich
mete.soner@math.ethz.ch

Abstract

As well known in the optimal transport problem, given two measures of equal mass, we look for an optimal map that takes one measure to the other one and also minimizes a given cost functional. In robust hedging problems, we are also given two measures. Namely, the initial and the final distributions of a stock process. We then construct an optimal connection. In general, however, the cost functional depends on the whole path of this connection and not simply on the final value. Hence, one needs to consider processes instead of simply the transport maps. The probability distribution of this process has prescribed marginals at final and initial times. Thus, it is in direct analogy with the Kantorovich measure. But, financial considerations restrict the process to be a martingale. Interestingly, the dual also has a financial interpretation as a robust hedging (super-replication) problem.

In this talk, we prove an analogue of Kantorovich duality: the minimal super- replication cost in the robust setting is given as the supremum of the expectations of the contingent claim over all martingale measures with a given marginal at the maturity. This joint work with Yan Dolinsky from Hebrew University.

Wednesday 9:30am-10:30am

Initial-boundary value problems for transport equations with rough coefficients

Laura Spinolo

Istituto di Matematica Applicata e Tecnologie Informatiche, Pavia
spinolo@imati.cnr.it

Abstract

I will be concerned with existence and uniqueness results for transport equations with weakly differentiable coefficients. The fundamental papers by Di Perna and Lions and by Ambrosio establish well-posedness of the Cauchy problem for transport equations with Sobolev and BV (bounded total variation) coefficients, respectively. This analysis has relevant applications to the study of several nonlinear partial differential equations, like for instance hyperbolic systems of conservation laws in several space dimensions.

My talk will aim at discussing existence and uniqueness results concerning solutions of initial-boundary value problems for transport equations with BV (bounded total variation) coefficients. I will also exhibit counter-examples showing that, as soon as the BV regularity deteriorates at the domain boundary, uniqueness is in general violated.

The talk will be based on joint works with G. Crippa and C. Donadello.

Monday 5pm-6pm

On the Hardy constant of some non-convex planar domains

Achilleas Tertikas

Department of Mathematics and Applied Mathematics, University of Crete
tertikas@math.uoc.gr

Abstract

The Hardy constant of a simply connected domain $\Omega \subset \mathbb{R}^n$ is the best constant for the inequality

$$\int_\Omega |\nabla u|^2 dx \geq c \int_\Omega \frac{u^2}{\mathrm{dist}(x, \partial\Omega)^2}\, dx\ , \qquad u \in C_c^\infty(\Omega).$$

After the work of Ancona where the universal lower bound 1/16 was obtained, there has been a substantial interest on computing or estimating the Hardy constant of planar domains.

In this talk I will review on some recent work, on computing Hardy constants of non convex planar simply connected domains.

Monday 12pm-1pm