Adaptive multilevel solvers for the discontinuous Petrov-Galerkin method with an emphasis on high-frequency wave propagation problems

This work focuses on the development of fast and efficient solution schemes for the simulation of challenging problems in wave propagation phenomena. In particular, emphasis is given in high frequency acoustic and electromagnetic problems which are characterized by localized solutions. This kind of simulations are essential in various applications, such as ultrasonic testing, laser scanning and modeling of optical laser amplifiers.

In wave simulations, the computational cost of any numerical method, is directly related to the frequency. In the high-frequency regime very fine meshes have to be used in order to satisfy the Nyquist criterion and overcome the pollution effect. This often leads to prohibitively expensive problems. Numerical methods based on standard Galerkin discretizations, lack of pre-asymptotic discrete stability and therefore adaptive mesh refinement strategies are usually inefficient. Additionally, the indefinite nature of the wave operator makes state of the art preconditioning techniques, such as multigrid, unreliable.

In this work, a promising alternative approach is followed within the framework of the discontinuous Petrov-Galerkin (DPG) method. The DPG method offers numerous advantages for our problems of interest. First and foremost, it offers mesh and frequency independent discrete stability even in the pre-asymptotic region. This is made possible by computing on the fly an optimal test space as a function of the trial space. Secondly, it provides a built-in local error indicator that can be used to drive adaptive refinements. Combining these two properties together, reliable adaptive refinement strategies are possible which can be initiated from very coarse meshes. Lastly, the DPG method can be viewed as a minimum residual method, and therefore it always delivers symmetric (Hermitian) positive definite stiffness matrix. This is a desirable advantage when it comes to the design of iterative solution algorithms. Conjugate Gradient based solvers can be employed which can be accelerated by domain decomposition (one- or multi- level) preconditioners for symmetric positive definite systems.

Driven by the aforementioned properties of the DPG method, an adaptive multigrid preconditioning technology is developed that is applicable for a wide range of boundary value problems. Unlike standard multigrid techniques, our preconditioner involves trace spaces defined on the mesh skeleton, and it is suitable for adaptive \( hp \)-meshes. Integration of the iterative solver within the DPG adaptive procedure
turns out to be crucial in the simulation of high frequency wave problems. A collection of numerical experiments for the solution of linear acoustics and Maxwell equations demonstrate the efficiency of this technology, where under certain circumstances, uniform convergence with respect to the mesh size, the polynomial order and the frequency can be achieved. The construction is complemented with theoretical estimates for the condition number in the one-level setting.