

## Κεφάλαιο 1

4.  $a = -1, b = -2, c = 1$

5. (i)  $y = \frac{2}{13}x^{\frac{13}{2}} + \frac{10}{3}x^{\frac{3}{2}} + c$  (ii)  $y = -5e^{-x} + c$   
(iii)  $y = -\frac{1}{2}x^4 + x^2 + c_1x + c_2$  (iv)  $y = x^3 + x + \frac{c}{x}$ .

6. (i)  $c = 7$  (ii)  $c = -\pi$  (iii)  $c = \frac{\pi}{4}$  (iv)  $c = -56$

7. (i)  $\dot{\mathbf{r}} = (6 - 2e^{-t})\mathbf{i} + (5 \sin t - 3)\mathbf{j} + (3 \cos t - 1)\mathbf{k}$ ,  $\mathbf{r} = (6t + 2e^{-t} - 1)\mathbf{i} + (2 - 5 \cos t - 3t)\mathbf{j} + (3 \sin t - t + 2)\mathbf{k}$

(ii)  $\dot{\mathbf{r}} = (4t^3 + 6)\mathbf{i} + (9t^2 - 8t + 15)\mathbf{j} - (3t^2 + 8)\mathbf{k}$ ,  $\mathbf{r} = (t^4 + 6t + 3)\mathbf{i} + (3t^3 - 4t^2 + 15t - 1)\mathbf{j} + (4 - t^3 - 8t)\mathbf{k}$

## Κεφάλαιο 2

1. (i)  $(e^x + 1)^{-2} + 2(e^y + 1)^{-1} = c$  (ii)  $(y + 1)^{-1} + \ln |y + 1| = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + c$

(iii)  $\left(\frac{y+3}{x+4}\right)^5 = ce^{y-x}$  (iv)  $y = \cot^{-1}(c - \cos x)$

(v)  $y = \sin\left(\frac{x^2}{2} + c\right)$  (vi)  $y = \frac{1}{c - \tan^{-1}(e^x)}$

2. (i)  $(x + 1)^6(y^2 + 1) = c(x + 2)^4$  (ii)  $y^2 + xy = cx^3$

(iii)  $\sin \frac{y}{x} = cx$  (iv)  $(x^2 + y^2)^{\frac{3}{2}} = x^3 \ln cx^3$

3. (i)  $y(x) = -3 + \sqrt{1 + 2x}$  (ii)  $y(x) = \sin^{-1}(x^2 + \frac{1}{2})$

(iii)  $4y^5 + 20y^2 = 5(2x - 1)e^{2x} + 21$  (iv)  $y(x) = \tan\left(\frac{2x^2 - 12x + \pi}{4}\right)$

4. (i)  $(x - y) \ln |x - y| = x + c(x - y)$  (ii)  $x + y \ln |x| = cy$

(iii)  $\ln(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} = c$  (iv)  $4x = y(\ln |y| - c)^2$

5. (i)  $2\frac{y}{x} + \ln \left| \frac{y}{2x^2} \right| = 4$  (ii)  $y = \frac{x^3 + x}{2}$

(iii)  $y^3 + 3x^3 \ln |x| = 8x^3$  (iv)  $\ln |x| = e^{\frac{y}{x}} - 1$

6. (i)  $y = 1 + ce^{\frac{1}{x}}$  (ii)  $3(x^2 + x)y = x^3 - 3x + c$

(iii)  $y = \frac{1 + ce^{-x}}{x}$  (iv)  $y = (x + c) \cos x$

(v)  $2(1 + \sin x)y = x + \sin x \cos x + c$  (vi)  $y = (x^3 + c)e^{-3x}$

7. (i)  $y = x^4 - 2x^2$  (ii)  $y = (e^x + 1)^2$

(iii)  $y = \frac{1}{2} \sin 2x + \frac{\sqrt{2}}{2} \cos x$  (iv)  $y = \frac{1}{5}(2e^x - \sin 2x - 2 \cos 2x)$

8. (i)  $y = \frac{x}{x+c}$  (ii)  $y = \sqrt{2 + cx^{-1}e^{-x}}$

(iii)  $y^3 = 1 + cx^{-3}$  (iv)  $y^{-3} = x + \frac{1}{3} + ce^{3x}$

9. (i)  $x^2y^4 = x^4 + 15$  (ii)  $\frac{1}{\sqrt{xy}} = -\frac{1}{2}x + 1$

(iii)  $y = \frac{1}{\sqrt{2e^{2x} - 1}}$  (iv)  $y = \frac{1}{4}e^{-2x}$

10. (i)  $y = (x - 2 + ce^{-x})^{-1} + 1$  (ii)  $\frac{e^{-\frac{x^2}{2}}}{y-x} = \int e^{-\frac{x^2}{2}} dx + c$

(iii)  $y = (2 + ce^{-2x^2})^{-1} + x$

11. (i)  $y(x) = \frac{3x+2}{2x}$  (ii)  $y(x) = \frac{x^3-x-1}{x^2+x}$

(iii)  $y(x) = 2 + \frac{1}{e^x + \sin x}$

12. (i)  $x^2y + x + 2y^2 = c$  (ii)  $3x^2y + 2xy^2 - 5x - 6y = c$

(iii)  $(y^2 + 1) \sin x = c$  (iv)  $y \tan x + \sec x + y^2 = c$

13. (i)  $x^3y^2 - xy^3 + x^2 + y + 1 = 0$  (ii)  $y^2 \cos x - y \sin^2 x = 9$

(iii)  $e^x y + xy^2 + 2e^x = 8$  (iv)  $2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 4x^{\frac{4}{3}}y^{\frac{1}{3}} = 9$

14.  $\mu = x^2, x^4 + x^3y^2 = c$

- 15.** (i)  $\mu = |x^3|$ ,  $4x^5y + 4x^4y^2 + x^4 = c$       (ii)  $\mu = |\cos x|$ ,  $x^2 \cos y + x \sin y = c$   
 (iii)  $\mu = e^x$ ,  $xy^2e^x + ye^x = c$       (iv)  $\mu = y^{-2}$ ,  $x^2 + xy^{-1} + y^2 = c$
- 16.** (i)  $\mu = x^2y^3$ ,  $x^3y^4(xy + 2) = c$       (ii)  $\mu = x^{-\frac{1}{3}}y^{-\frac{4}{3}}$ ,  $x^{\frac{2}{3}}y^{\frac{5}{3}}(x^2 - y) = c$
- 17.** (i)  $5x^2 + 4xy + y^2 + 2x + 2y = c$       (ii)  $x - 2y + \ln |3x - y - 2| = c$   
 (iii)  $\ln[c(x^2 + y^2 - 2x + 2y + 2)] + 4 \tan^{-1}(\frac{y+1}{y-1}) = 0$       (iv)  $(2x + y + 3)^3(x - y + 1)^2 = c$
- 18.** (i)  $\ln[3(x - 1)^2 + (y + 3)^2] + \frac{2}{\sqrt{3}} \tan^{-1} \frac{y+3}{\sqrt{3}(x-1)} = \ln 4 + \frac{\pi}{3\sqrt{3}}$   
 (ii)  $x + 2y - 2 - \ln |2x + 3y - 1| = 0$
- 19.** (i)  $y = cx + \frac{1}{c^2}$ ,  $4y^3 = 27x^2$       (ii)  $x = \frac{2p}{p^2-1}$ ,  $y = \frac{2}{p^2-1} - \ln |p^2 - 1| + c$   
 (iii)  $y^2 = 2cx + c^3$ ,  $32x^3 + 27y^4 = 0$ ,  $y = 0$       (iv)  $2x = 1 + 2 \ln |y|$ ,  $y^2 = 2cx - c \ln c$   
 (v)  $x = \frac{\ln p+c}{\sqrt{p}}$ ,  $y = \sqrt{p}(4 - \ln p - c)$ ,  $y = 0$
- 20.**  $u' - Q(x)u = -P(x)$ ,  $y = e^{x^2+cx^{-2}}$
- 21.** (i)  $\sin^2 y = 4x^2 - cx$       (ii)  $e^{2y} - x^2 + 2xe^y = c$   
 (iii)  $y = \left(6x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} + c\right)^{-1}$       (iv)  $y = \frac{1}{3t+ct^2} + \frac{1}{t}$   
 (v)  $y^2 = c_1e^x + c_2$
- 22.** (i)  $\ln |x+y| + \frac{x}{x+y} = c$ ,  $y = -x$       (ii)  $8y - 4x + \ln |4x + 8y + 5| = c$ ,  $4x + 8y + 5 = 0$   
 (iii)  $ye^x(2x - y) = c$       (iv)  $y = (1+x)(\frac{1}{2}x^2 + c)$   
 (v)  $x^2y^2 = -2x \cos x + 2 \sin x + c$       (vi)  $4y = x^2$ ,  $5y = -5x^2 + 5cx - c^2$
- 23.** (i)  $x^2 + 3y^2 = k62$       (ii)  $y^2(\ln y - \frac{1}{2}) = -x^2 + k$   
 (iii)  $x^2 + y^2 - 2x + 4 \ln(x + 2) = k$       (iv)  $x^2y = \frac{1}{4}x^4 + k$   
 (v)  $y = k(x^2 + 3y^2)$
- 24.** (i)  $\ln |2\sqrt{3}x^2 - xy + \sqrt{3}y^2| - \frac{6}{\sqrt{23}} \tan^{-1} \frac{2\sqrt{3}y-x}{\sqrt{23}x} = c$   
 (ii)  $\ln |3x^2 + 3xy + 4y^2| = -\frac{2}{\sqrt{39}} \tan^{-1} \left(\frac{3x+8y}{\sqrt{39}x}\right) = c$

### Κεφάλαιο 3

- 1.** (i)  $W = e^{5x} \neq 0$  (ii)  $y = c_1e^{2x} + c_2e^{3x}$  (iii)  $y = 3e^{2x} - re^{3x}$ ,  $x \in (-\infty, \infty)$
- 2.** (i)  $W = -\frac{4}{x} \neq 0$  (ii)  $y = c_1x^2 + c_2x^{-2}$  (iii)  $y = \frac{1}{4}x^2 + 8x^{-2}$ ,  $x \in (0, \infty)$
- 3.** (i)  $y = c_1x + c_2(x - 1)e^x$  (ii)  $y = c_1e^{2x} + c_2(x + 1)$  (iii)  $y = c_1e^x + c_2x^{-1}$
- 4.** (i)  $y = c_1e^{\frac{x}{2}} + c_2e^{-2x}$       (ii)  $y = (c_1x + c_2)e^{\frac{x}{2}}$   
 (iii)  $y = e^{-x}(c_1 \sin \frac{3}{4}x + c_2 \cos \frac{3}{4}x)$       (iv)  $y = (c_1x + c_2)e^{4x}$   
 (v)  $y = e^{2x}(c_1 \sin 3x + c_2 \cos 3x)$       (vi)  $y = c_1 \sin \frac{1}{2}x + c_2 \cos \frac{1}{2}x$
- 5.** (i)  $y = 2e^{4x} + e^{-3x}$       (ii)  $y = (2 + 3x)e^{-3x}$   
 (iii)  $y = e^{2x} \sin 5x$       (iv)  $y = e^{-\frac{x}{2}}(2 \cos 3x - \sin 3x)$
- 6.** (i)  $y = c_1e^x + c_2e^{2x} + 2x^2 + 6x + 7$       (ii)  $y = e^{-x}(c_1 \sin x + c_2 \cos x) - \frac{7}{13} \sin 4x - \frac{4}{13} \cos 4x$   
 (iii)  $y = e^{-x}(c_1 \sin 3x + c_2 \cos 3x) + (\frac{1}{2}x + \frac{1}{10})e^{-2x}$       (iv)  $y = c_1 \cos 2x + c_2 \sin 2x + 2x \sin 2x - x \cos 2x$   
 (v)  $y = c_1e^{2x} + c_2e^{-2x} + 2x^2e^{2x} - xe^{2x}$       (vi)  $y = c_1 \sin x + c_2 \cos x - \frac{1}{4}x^2 \cos x + \frac{1}{4}x \sin x$
- 7.** (i)  $y = 3e^{3x} - 2e^{5x} + 3xe^{2x} + 4e^{2x}$       (ii)  $y = (3x - 5)e^{-3x} + 3e^{-6x}$   
 (iii)  $y = 2e^{5x}(2 \sin 2x - \cos 2x + 1)$       (iv)  $y = (3x^2 + x + 5)e^x + (2x - 4)e^{2x}$
- 8.** (i)  $y = c_1 \sin x + c_2 \cos x + x \sin x + \cos x \Upsilon |\cos x|$       (ii)  $y = e^{-2x}(c_1 \sin x + c_2 \cos x) + xe^{-2x} \sin x + e^{-2x} \cos x \ln |\cos x|$   
 (iii)  $y = (c_1x + c_2)e^x - \frac{5}{36}x^3e^x + \frac{1}{6}x^3e^x \ln x$       (iv)  $y = c_1 \sin x + c_2 \cos x + \frac{1}{2} \tan x + \frac{3}{2} \cos x \ln |\sec x + \tan x|$

$$(v) y = c_1 e^{-x} + c_2 e^{-2x} + (e^{-x} + e^{-2x}) \ln(1 + e^x) \quad (vi) y = c_1 e^{-x} + c_2 e^{-2x} + e^{-x} \ln|x| - e^{-2x} \int \frac{e^x}{x} dx$$

$$9. (i) y = c_1 x + c_2 x e^x - x^2$$

$$(ii) y = c_1 x + \frac{c_2}{x+1} + x^2 - \frac{2x^3+3x^2}{6(x+1)}$$

$$(iii) y = c_1 \sin x + c_2 x \sin x + \frac{1}{2} x^2 \sin x$$

$$10. (i) y = c_1 x^{\frac{1}{2}} + c_2 x^{\frac{3}{2}} \quad (ii) y = x^2 [c_1 \sin(\ln x^3) + c_2 \sin(\ln x^3)]$$

$$(iii) y = (c_1 + c_2 \ln x) x^{\frac{1}{3}} \quad (iv) y = c_1 x^2 + c_2 x^4 - 2x^3$$

$$(v) y = c_1 \sin(\ln x^2) + c_2 \cos(\ln x^2) + \frac{1}{5} x \ln x^2 - \frac{4}{25} x \quad (vi) y = x^2 [c_1 \sin(\ln x) + c_2 \cos(\ln x)] + 5x^2$$

$$11. (i) y = 2x^5 + 3x^{-2} \quad (ii) y = x^{-1} + x^2 - 2x + 4$$

$$(iii) y = \frac{5}{3} x - 2x^2 + 3x^3 - \frac{23}{24} x^4 \quad (iv) y = \frac{1}{18} x^3 + \frac{1}{12} x^{-2} - \frac{1}{6} \ln x + \frac{1}{36}$$

$$12. y = c_1 (x+2)^3 + c_2 (x+2)^{-1}$$