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Short communication

Incompressible Poiseuille flows of Newtonian liquids with a pressure-dependent viscosity

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ABSTRACT

The pressure-dependence of the viscosity becomes important in flows where high pressures are encountered. Applications include many polymer processing applications, microfluidics, fluid film lubrication, as well as simulations of geophysical flows. Under the assumption of unidirectional flow, we derive analytical solutions for plane, round, and annular Poiseuille flow of a Newtonian liquid, the viscosity of which increases linearly with pressure. These flows may serve as prototypes in applications involving tubes with small radius-to-length ratios. It is demonstrated that, the velocity tends from a parabolic to a triangular profile as the viscosity coefficient is increased. The pressure gradient near the exit is the same as that of the classical fully developed flow. This increases exponentially upstream and thus the pressure required to drive the flow increases dramatically.

1. Introduction

The viscosity of fluids, such as polymer melts and lubricants, depends strongly on temperature and to a less extent to pressure [1]. In such fluids, the dependence of the viscosity on pressure may be several orders of magnitude stronger than that of density [1,2]. Denn [3] emphasized that at a pressure of about 5 MPa, which can be reached in extrusion and in injection molding, the pressure dependence of the viscosity is expected to become important while the flow is still incompressible. Therefore, it is reasonable to study isothermal, incompressible flow of fluids with a pressure-dependent viscosity.

The idea of a fluid with pressure-dependent viscosity was introduced by Stokes [4]. Barus [5] proposed an exponential isothermal equation of state for the viscosity of the form

$$\eta(p) = \eta_0 e^{\beta p}, \tag{1}$$

where η is the viscosity, p is the pressure, η_0 is the viscosity at atmospheric pressure, and β is the pressure-viscosity coefficient (which is temperature dependent). In polymer melts, β is typically $1\text{–}5 \times 10^{-8}\ \text{Pa}^{-1}$ [3]. For lubricants, β varies from 10 to 70 MPa^{-1} [6]. Venner and Lubrecht [7] reported that for mineral oils β is generally in the range between 10^{-8} and $2 \times 10^{-8}\ \text{Pa}^{-1}$. Carreras et al. [8] compiled experimental values of the shear pressure coefficient β. Even though Eq. (1) is extensively used, it is valid as a reasonable approximation only at moderate pressures. A compilation of other equations proposed for the pressure dependence of the viscosity and useful references on the subject has been provided by Málek and Rajagopal [9].

Numerous are the experimental studies concerning the determination of the pressure dependence of the viscosity of common polymer grades, such as polyethylenes (LDPE, LLDPE, HDPE), polypropylene and polystyrene. Comprehensive reviews are provided by Binding et al. [10] and Goubert et al. [11] who compared measurement techniques in the literature for evaluating the pressure dependence of viscosity.

As already mentioned high pressures sufficient to cause significant change in the viscosity appear in many polymer processing operations. Driving pressures of 50 and 100 MPa are routinely required in extrusion and injection molding [12]. The strong effect of pressure and its potential importance in plastics processing led to the development of high-pressure rheometers based on pressure driven or drag flow [13]. Cardinaels et al. [14] discussed different methods to obtain pressure coefficients for different polymers, such as PMMA and LDPE, from high-pressure capillary rheometer data. More recently, Park et al. [15] also compared different experimental methods for the determination of the pressure coefficient of a styrenic polymer.

The pressure-dependence of the viscosity becomes important in other applications, such as fluid film lubrication, microfluidics, and geophysics. In fluid film lubrication studies it is essential to include the variation of the viscosity with pressure [16]. For technological applications in elastohydrodynamic lubrication and in thrust bearing or journal bearing applications, where the lubricant is forced

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to flow through a very narrow region which leads to very high pressures, the reader is referred to the work of Gwynllyw et al. [17]. In the design of Micro Electro-Mechanical Systems (MEMS), the pressure-dependence of the viscosity needs to be taken into account. Experimental data for liquid flows in microtubes driven by high pressures (1–30 MPa) show that the pressure gradient is not constant, an effect attributed to the pressure-dependence of the viscosity [18,19]. In geophysical flows, the viscosity changes with the depth of the fluid. Convection in planetary mantles is most likely dominated by the strong variability of the mantle viscosity depending on temperature and pressure [20]. In her mantle flow simulations, Georgen [21] allowed the viscosity to vary over three orders of magnitude from 10^{19} to 10^{22} Pa s.

Mathematical issues arising in the case of incompressible Newtonian or non-Newtonian flows with a pressure-dependent viscosity have been addressed by Renardy [22], Gazzola [23], and Málek et al. [24,25]. The existence of flows of fluids with pressure-dependent viscosity and the associated assumptions have been discussed by Bulíček et al. [26]. The properties of such solutions are also discussed by Málek and Rajagopal [9].

In addition to Eq. (1), Hron et al. [27] also assumed the following expression for the viscosity pressure dependence:

$$\eta(p) = \beta p. \tag{2}$$

They showed that unidirectional flows are not possible between parallel plates in the case of the former model, since a secondary flow is necessary to that end. However, unidirectional flows are possible in the latter case.

Renardy [2] considered parallel shear flows of an incompressible Newtonian fluid allowing a general pressure dependence for the viscosity and proved that a sufficient condition for the existence of parallel pressure-driven flow in a pipe, regardless of its cross-section, is the linear dependence of the viscosity on the pressure:

$$\eta(p) = \eta_0(1 + \beta p). \tag{3}$$

This condition is not necessary; Denn [28] showed that the quadratic velocity profile in a circular pipe remains a solution if the viscosity is an exponential function of the pressure. As indicated by Renardy [2] and also shown in the present work, the velocity profile is not parabolic in the case of linear dependence of the viscosity; it may be almost parabolic when this dependence is weak. According to Suslov and Tran [29], the major concern of linear constitutive equation (3) is that it does not guarantee positive definiteness of the viscosity which requires the pressure to remain positive. This problem is not encountered when using exponential constitutive equation (1) or in flows where the pressure remains positive, such as Poiseuille flows.

It seems that Eq. (2) has been the most popular one in the various theoretical analyses presented in the literature. Analytical solutions have been reported by Renardy [2] and Vasudevaiah and Rajagopal [30] for the round Poiseuille flow of a Newtonian fluid and by Hron et al. [27] and Huilgol and You [31] for the plane Poiseuille flow of a generalized Newtonian fluid. The reason of avoiding Eq. (1) is obvious, since this equation rules out the possibility of having analytical solutions, but Eq. (3) should be more preferable than Eq. (2), since the latter predicts a vanishing viscosity at zero pressure. Another advantage of Eq. (3) over Eq. (2) is that it involves a reference viscosity constant. However, as shown below, both equations result in the same solution for the velocity in the case of unidirectional Poiseuille flow. What is different is the pressure distribution.

In the present work, we derive and discuss analytical solutions of axisymmetric, annular, and plane Poiseuille flows of Newtonian fluids with pressure-dependent viscosity obeying Eq. (3).

The rest of the paper is organized as follows: in Section 2 the governing equations are presented and the derivation of the analytical solution is described in the case of the round Poiseuille flow. The solutions for the other two Poiseuille flows of interest are also provided. In Section 3, the theoretical results and the effects of the viscosity pressure-dependence are discussed. Finally, in Section 4 we provide the conclusions and some suggestions for future work.

2. Governing equations and analytical solutions

For an incompressible Newtonian fluid, the viscosity of which is a function of pressure, the viscous stress tensor is given by

$$\boldsymbol{\tau} = 2\eta(p)\mathbf{D}, \tag{4}$$

where

$$\mathbf{D} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] \tag{5}$$

is the rate-of-deformation tensor and $\boldsymbol{u}$ is the velocity vector. It can be shown in this case that the Navier–Stokes equation in the absence of gravity becomes:

$$\rho\left(\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \eta(p)\nabla^2\mathbf{u} + 2\eta'(p)\nabla p\cdot\mathbf{D}. \tag{6}$$

It should also be noted that the continuity equation for incompressible flow is

$$\nabla\cdot\mathbf{u} = 0. \tag{7}$$

In this paragraph we consider incompressible Poiseuille flows of Newtonian fluids with pressure-dependent viscosity obeying Eq. (3).

2.1. Axisymmetric Poiseuille flow

We consider the nondimensionalized governing equations of axisymmetric Poiseuille flow in cylindrical coordinates with the origin located at the exit of the tube. The radial coordinate, r, is scaled by the radius R and the axial coordinate, z, by the length L of the tube. Moreover the axial velocity is scaled by the mean velocity U, the pressure by $8\eta_0 LU/R^2$ (chosen so that the pressure at the inlet plane is equal to 1), and the viscosity η by η_0. Hence the dimensionless form of the viscosity equation becomes

$$\eta^* = 1 + \varepsilon p^*, \tag{8}$$

where stars denote dimensionless quantities and

$$\varepsilon \equiv \frac{8\beta\eta_0 LU}{R^2}. \tag{9}$$

For convenience, stars will be dropped hereafter. Under the assumption that the radial velocity component is zero, the continuity equation dictates that $u_z = u_z(r)$; hence, only pressure is a function of both r and z, $p = p(r,z)$. As pointed out by Huilgol and You [31], it is clear that as long as $\partial\eta/\partial p$ is nonzero, a pressure gradient in the flow direction induces one in the direction of the velocity gradient, unless inertia is present. The z- and r-components of the momentum equation, defined over the domain $[0,1]\times[-1,0]$, are simplified as follows:

$$-8\frac{\partial p}{\partial z} + (1+\varepsilon p)\frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right) + \varepsilon\frac{\partial p}{\partial r}\frac{du_z}{dr} = 0 \tag{10}$$

and

$$-8\frac{\partial p}{\partial r} + \varepsilon\alpha^2\frac{\partial p}{\partial z}\frac{du_z}{dr} = 0, \tag{11}$$

where

$$\alpha \equiv \frac{R}{L} \tag{12}$$

is the tube aspect ratio. By eliminating $\partial p/\partial r$ and separating variables we find that

$$\frac{(1/r)(du_z/dr) + (d^2u_z/dr^2)}{1 - (\varepsilon^2\alpha^2/64)(du_z/dr)^2} = \frac{8}{1+\varepsilon p}\frac{\partial p}{\partial z} = -A \tag{13}$$

where A is in general a function of r, taken here as a constant to be determined. We have thus two differential equations to be solved for u_z and p. By solving the first equation for u_z and applying the symmetry boundary condition at the axis of symmetry and the no-slip condition at $r = 1$, one finds that

$$u_z(r) = \frac{64}{A\varepsilon^2\alpha^2} \ln\left[\frac{I_0(A\varepsilon\alpha/8)}{I_0((A\varepsilon\alpha/8)r)}\right], \tag{14}$$

where I_0 is the modified Bessel function of kind one and zero order [32]. The above expression has been previously derived by Renardy [2] and Vasudevaiah and Rajagopalan [30] who employed Eq. (2) instead of Eq. (3). By integrating the other differential equation, assuming that $p(0,0) = 0$, and taking into account the velocity profile, we find that

$$p(r,z) = \frac{1}{\varepsilon}\left[I_0\left(\frac{A\varepsilon\alpha r}{8}\right)e^{-A\varepsilon z/8} - 1\right]. \tag{15}$$

The constant A is determined by demanding that the volumetric flow rate is 2π. This leads to the following equation

$$2\int_0^1 \ln\left[I_0\left(\frac{A\varepsilon\alpha r}{8}\right)\right] r dr - \ln\left[I_0\left(\frac{A\varepsilon\alpha}{8}\right)\right] + \frac{A\varepsilon^2\alpha^2}{64} = 0, \tag{16}$$

which is easily solved for A by means of Newton's method combined with numerical integration.

If instead of Eq. (3), the following equation is used, as was done by Hron et al. [27],

$$\eta(p) = \varepsilon p, \tag{17}$$

the above procedure leads to Eq. (15) for the velocity and to the expression

$$p(r,z) = \frac{1}{\varepsilon} I_0\left(\frac{A\varepsilon\alpha r}{8}\right)e^{-A\varepsilon z/8} \tag{18}$$

for the pressure. In both cases, the pressure increases exponentially upstream, which means that an enormous pressure drop may be achieved with a tube of finite length.

2.2. Annular Poiseuille flow

Let us now consider the Poiseuille flow in an annulus of radii κR and R, where $0 < \kappa < 1$. Using the same scalings and assumptions as in the axisymmetric case, we end up with the same separated differential equations to be solved for $u_z(r)$ and $p(r,z)$. An additional dimensionless number is introduced, i.e. the radii ratio κ. With the assumption of no slip along the two walls, the following expression is obtained for the slip velocity

$$u_z(r) = \frac{64}{A\varepsilon^2\alpha^2} \ln\left\{\frac{[K_0(B) - K_0(B\kappa)]\, I_0(B) - [I_0(B) - I_0(B\kappa)]\, K_0(B)}{[K_0(B) - K_0(B\kappa)]\, I_0(Br) - [I_0(B) - I_0(B\kappa)]\, K_0(Br)}\right\}, \tag{19}$$

where

$$B \equiv \frac{A\varepsilon\alpha}{8} \tag{20}$$

and K_0 is the modified Bessel function of the second kind of first order. Assuming that $p(\kappa,0) = 0$, the pressure is found to be given by

$$p(r,z) = \frac{1}{\varepsilon} \ln\left\{\frac{[K_0(B) - K_0(B\kappa)]\, I_0(Br) - [I_0(B) - I_0(B\kappa)]\, K_0(Br)}{[K_0(B) - K_0(B\kappa)]\, I_0(B) - [I_0(B) - I_0(B\kappa)]\, K_0(B)} e^{\varepsilon A z/8} - 1\right\}. \tag{21}$$

Assuming that the (dimensionless) volumetric flow is equal to 2π, we find that the constant A is the root of the following equation:

$$2\int_\kappa^1 \ln\left[(K_0(B) - K_0(B\kappa))I_0(Br) - (I_0(B) - I_0(B\kappa))K_0(Br)\right] \times r dr - (1-\kappa^2)\ln\left[(K_0(B) - K_0(B\kappa))I_0(B) - (I_0(B) - I_0(B\kappa))K_0(B)\right] + \frac{A\varepsilon^2\alpha^2}{64} = 0. \tag{22}$$

2.3. Plane Poiseuille flow

We consider the pressure-driven flow in a channel of half-width H and length L and work in Cartesian coordinates with the origin at the intersection of the midplane and the exit plane of the channel and the x-axis in the flow direction. We nondimensionalize the governing equations scaling x by L, y by H, u_x by the mean velocity U, and the pressure by $3\eta_0 LU/H^2$. The resulting dimensionless numbers are

$$\alpha \equiv \frac{H}{L} \quad \text{and} \quad \varepsilon \equiv \frac{3\beta\eta_0 LU}{H^2}. \tag{23}$$

One finds that the velocity and pressure are given by

$$u_x(y) = \frac{9}{A\varepsilon^2\alpha^2} \ln\left[\frac{\cosh(A\varepsilon\alpha/3)}{\cosh((A\varepsilon\alpha/3)y)}\right] \tag{24}$$

and

$$p(x,y) = \frac{1}{\varepsilon}\left[\cosh\left(\frac{A\varepsilon\alpha y}{3}\right)e^{-A\varepsilon x/3} - 1\right]. \tag{25}$$

The constant A is determined by demanding that the volumetric flow rate is equal to unity. It turns out that A is the root of

$$\int_0^1 \ln\left[\cosh\left(\frac{A\varepsilon\alpha y}{3}\right)\right] dy - \ln\left[\cosh\left(\frac{A\varepsilon\alpha}{3}\right)\right] + \frac{A\varepsilon^2\alpha^2}{9} = 0. \tag{26}$$

Solution (24) for the velocity has also been derived by Hron et al. [27] and Huilgol and You [31], who employed Eq. (2) for the pressure-dependence of the viscosity.

3. Discussion

In this section we discuss only results for the axisymmetric and annular Poiseuille flows (the results for the plane flow are similar to their axisymmetric counterparts). In order to construct solutions for the velocity and pressure for the axisymmetric Poiseuille flow, the constant A must be determined from Eq. (16). It turns out that the latter equation has a unique nonzero root only when the parameter

$$\alpha\varepsilon = \frac{8\beta\mu_0 U}{R} \tag{27}$$

is below the critical value $(\alpha\varepsilon)_{crit} = 8/3$. As illustrated in Fig. 1, at low values of $\alpha\varepsilon$, A is insensitive to $\alpha\varepsilon$; this is not the case at higher values and, as $\alpha\varepsilon$ approaches the critical value, A grows rapidly to infinity. In Fig. 2, the calculated velocity profiles for various values of the parameter $\alpha\varepsilon$ are shown. For $\alpha\varepsilon < 0.1$ the velocity has the

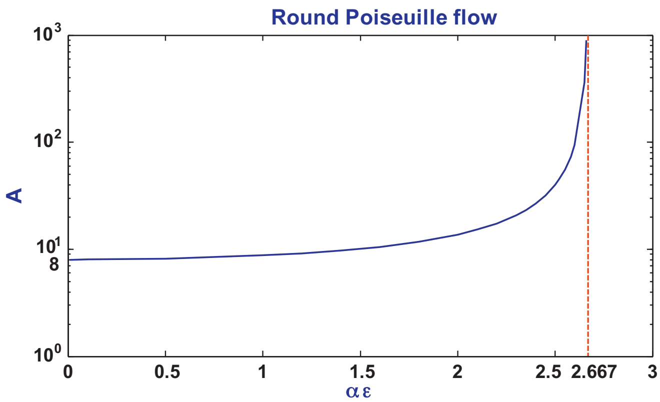

Fig. 1. The constant A as a function of the parameter $\alpha\varepsilon$ in axisymmetric Poiseuille flow.

parabolic profile for incompressible flow and then gradually tends to a linear profile:

$$u_{z,\text{crit}} = 3(1-r). \tag{28}$$

Let us point out that $(\alpha\varepsilon)_{\text{crit}}$ can be calculated analytically as the value zeroing the denominator of the LHS of Eq. (13). The velocity profiles of Fig. 2 suggest that in the two-dimensional flow the axial velocity is expected to change from a parabolic to a more triangular profile as we move upstream. The velocity profiles of Fig. 2 are essentially the same as those obtained by Renardy [2] and Vasudevaiah and Rajagopal [30] for a Newtonian fluid obeying Eq. (2) instead.

The pressure distributions obtained with $\alpha = 0.01$ and different values of $\alpha\varepsilon$ along the wall and the axis of symmetry are shown in Fig. 3. We observe that the pressure distribution remains linear only near the exit and that as the parameter $\alpha\varepsilon$ increases, the pressure upstream as well as the pressure gradient increase exponentially with the length of the tube. Clearly, the pressure required to drive the flow increases rapidly with the length of the tube. Assuming that this is given by $\Delta P = p(0,-1)$ and that $A \simeq 8$ is a reasonable approximation for sufficiently small values of $\alpha\varepsilon$, e.g. for very long tubes, one gets

$$\Delta P \approx \frac{1}{\varepsilon}(e^{\varepsilon}-1). \tag{29}$$

Now, if it is also assumed that ε is small, Eq. (30) gives

$$\Delta P \approx 1 + \frac{\varepsilon}{2} + \frac{\varepsilon^2}{6} + O(\varepsilon^3). \tag{30}$$

The above expression can be viewed as a correction factor for the Hagen–Poiseuille formula and can be used in measuring the vis-

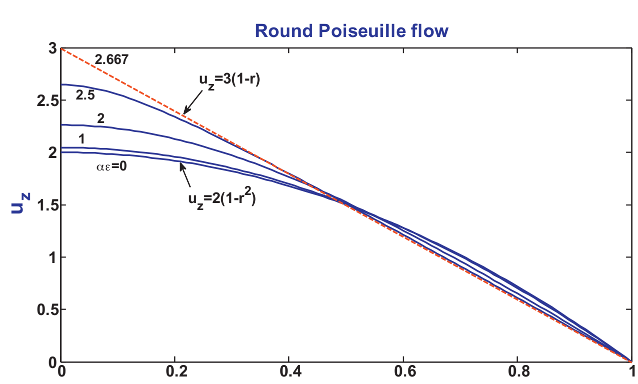

Fig. 2. Velocity profiles in axisymmetric Poiseuille flow for various values of the parameter $\alpha\varepsilon$.

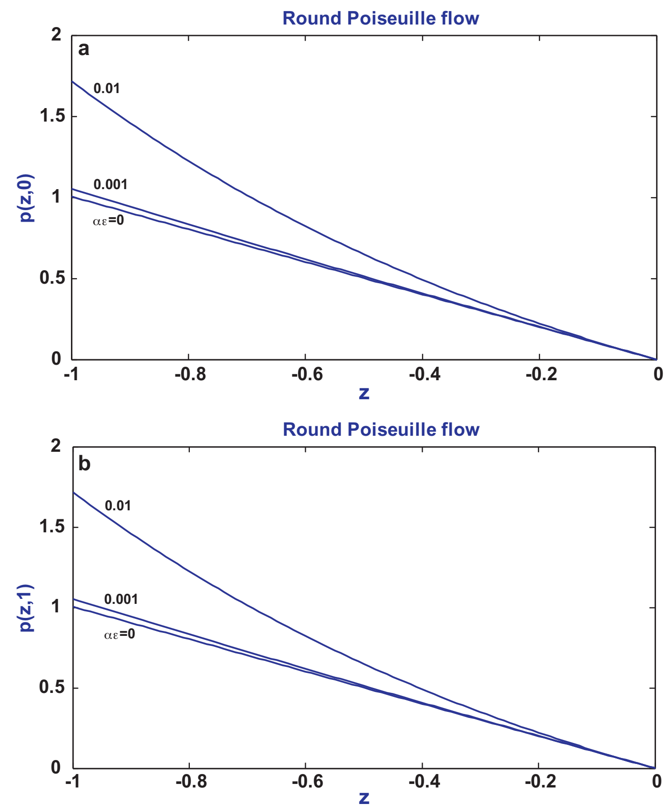

Fig. 3. Pressure distribution along (a) the axis of symmetry and (b) the wall, for $\alpha = 0.01$ and various values of $\alpha\varepsilon$; axisymmetric Poiseuille flow.

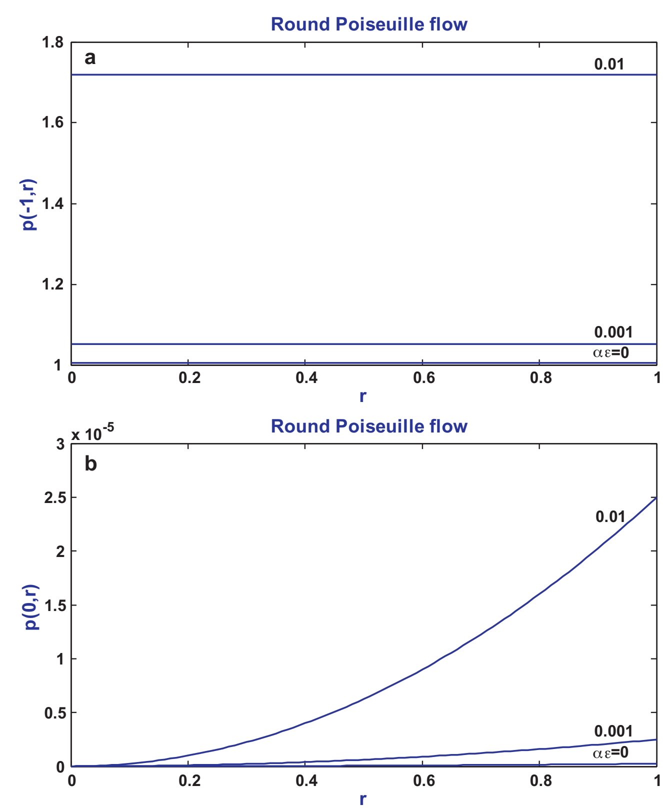

Fig. 4. Pressure distribution along (a) the inlet and (b) the outlet planes, for $\alpha = 0.01$ and various values of $\alpha\varepsilon$; axisymmetric Poiseuille flow.

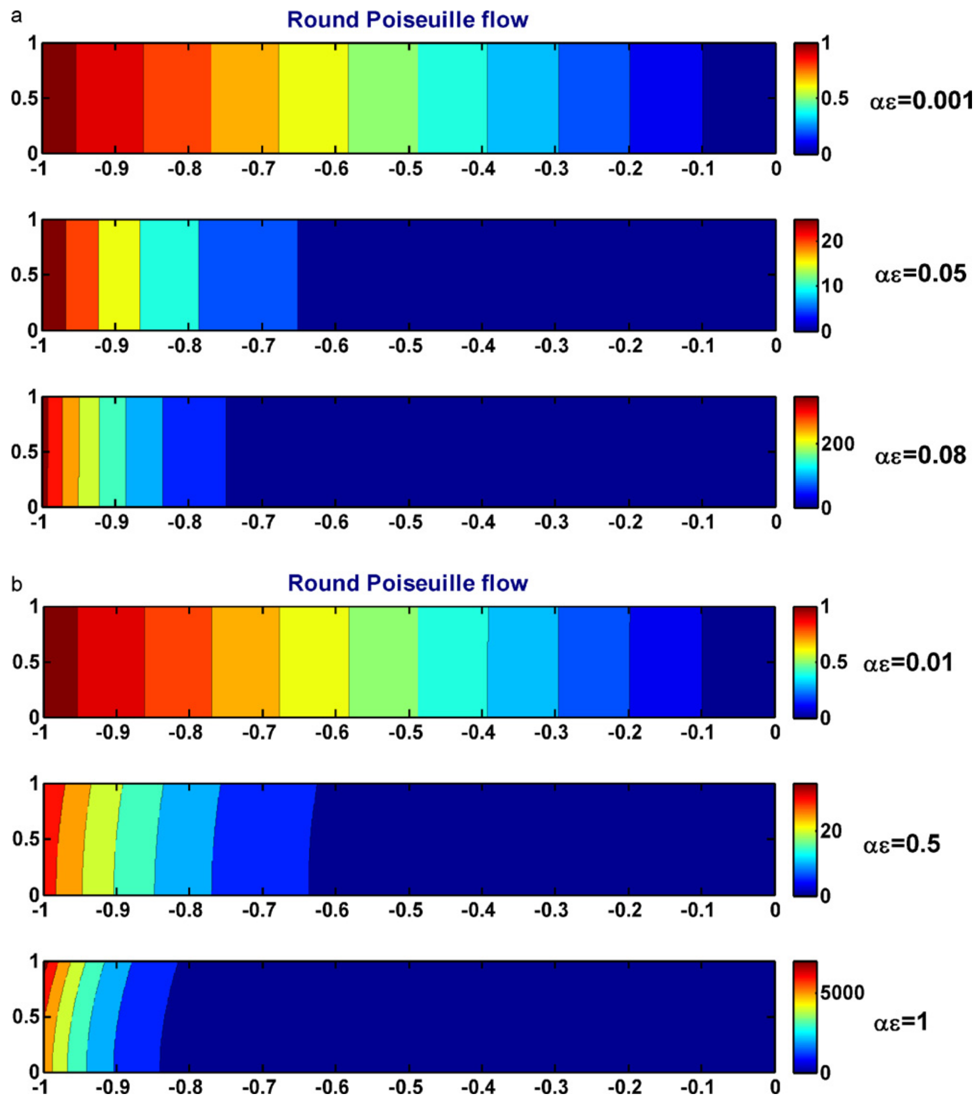

Fig. 5. Pressure contours for various values of $\alpha\varepsilon$ when (a) $\alpha = 0.01$ and (b) $\alpha = 0.1$; axisymmetric Poiseuille flow.

cosity from viscometric data obtained using capillaries of different length.

In Fig. 4, we show the pressure distributions along the inlet and outlet planes of the tube. We observe that the pressure starts deviating from the linear profile at sufficiently high values of $\alpha\varepsilon$. At the inlet plane the pressure seems to be insensitive to r, i.e. the relative deviations are negligible. This is not the case at the outlet plane where larger deviations are observed when moving from the axis of symmetry to the wall. However, the absolute value of pressure is essentially zero. These results are also illustrated in Fig. 5 where the pressure contours for a short ($\alpha = 0.1$) and a long ($\alpha = 0.01$) tube

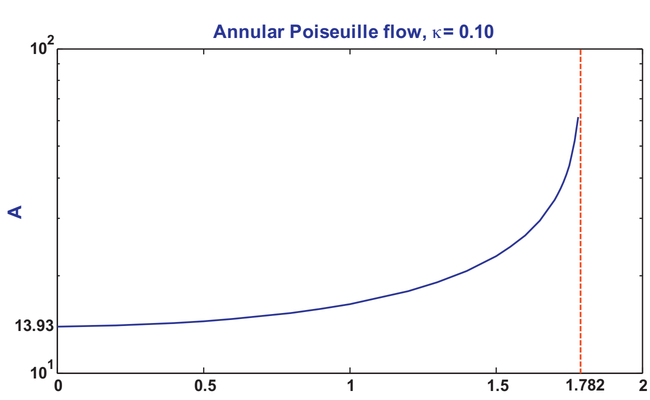

Fig. 6. The constant A as a function of the parameter $\alpha\varepsilon$ in annular Poiseuille flow for $\kappa = 0.1$.

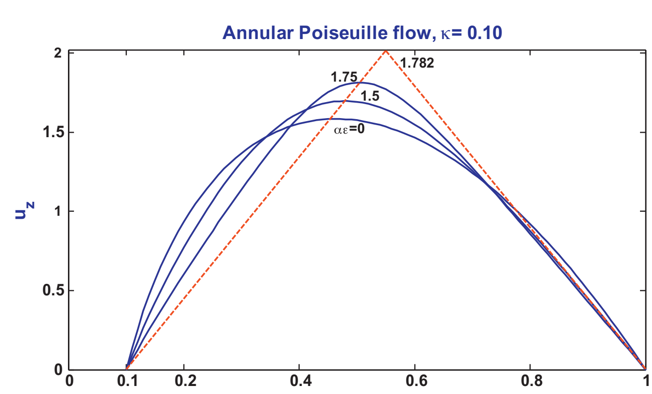

Fig. 7. Velocity profiles in annular Poiseuille flow for $\kappa = 0.1$, for various values of the parameter $\alpha\varepsilon$.

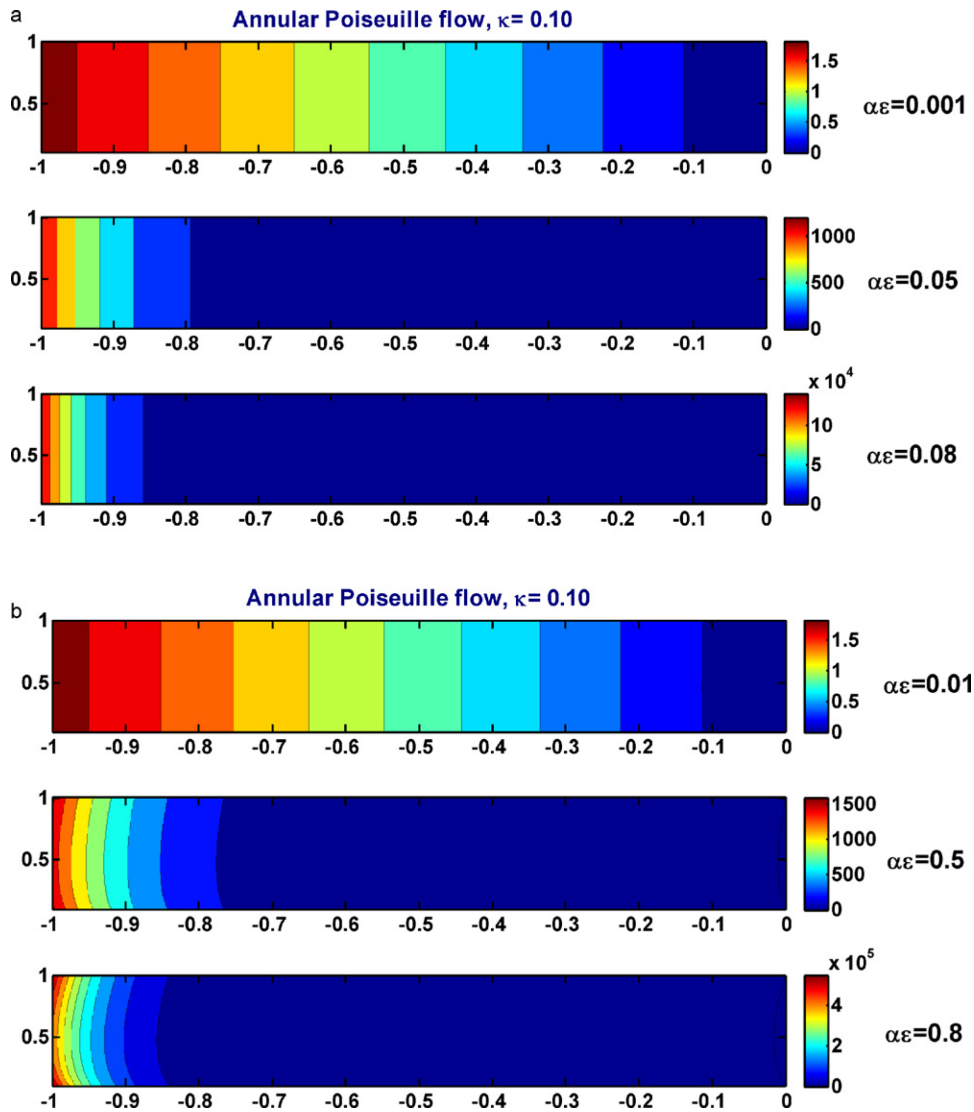

Fig. 8. Pressure contours for various values of $\alpha\varepsilon$ when (a) $\alpha = 0.01$ and (b) $\alpha = 0.1$; annular Poiseuille flow for $\kappa = 0.1$.

are plotted. For small values of α, the contours appear to be vertical; the bending of the contours is more clearly shown for bigger values of α, i.e. in shorter tubes.

In the case of annular Poiseuille flow, we have chosen to show results for $\kappa = 0.1$. In this case, the parameter A is a unique nonzero root of Eq. (22) when $\alpha\varepsilon$ is below the critical value 1.782, as illustrated in Fig. 6. It is easily shown that in general

$$(\alpha\varepsilon)_{\text{crit}} = 2(1+\kappa)(1-\kappa)^2 \tag{31}$$

and

$$u_{z,\text{crit}} = \begin{cases} \dfrac{4(r-\kappa)}{(1+\kappa)(1-\kappa)^2}, & \kappa \le r \le \dfrac{\kappa+1}{2} \\ \dfrac{4(1-r)}{(1+\kappa)(1-\kappa)^2}, & \dfrac{\kappa+1}{2} \le r \le 1 \end{cases}. \tag{32}$$

In Fig. 7, the velocity profiles for various values of the parameter $\alpha\varepsilon$ are shown. We notice that for $\alpha\varepsilon < 0.1$ the velocity has the parabolic profile for incompressible flow which steadily tends to the triangular profile described by Eq. (29) as $\alpha\varepsilon$ approaches the critical value.

As in round Poiseuille flow, the pressure gradient is roughly constant only for low values of $\alpha\varepsilon$. As the latter parameter increases, the pressure increases faster with the distance from the exit plane. Fig. 8 shows the pressure contours for a short ($\alpha = 0.1$) and a long ($\alpha = 0.01$) annulus and various values of $\alpha\varepsilon$. The vertical contours for small values of α begin to bend for bigger values of α, i.e. in shorter tubes.

4. Conclusions

Analytical solutions for the axisymmetric, annular, and plane Poiseuille flows of an incompressible Newtonian fluid with pressure-dependent viscosity, obeying Eq. (3), have been derived, under the assumption of unidirectional flow. These solutions show that as the pressure-dependence of the viscosity becomes stronger, the velocity profile, which is independent of the axial coordinate, tends from a parabolic-type to a triangular profile and the pressure, which is a function of both the axial and the radial coordinate, increases exponentially upstream. The latter result implies that the pressure required to drive the flow increases rapidly with the length of the tube.

The solution of the compressible Poiseuille flow of a Newtonian fluid with pressure-dependent viscosity is currently under investigation. Ideas for future work include the investigation of the combined effect of slip at the wall with viscosity pressure dependence and the solution of generalized Newtonian flows with pressure-dependent material parameters. Some interesting issues arise in the case of Bingham and other viscoplastic fluids with the exact definition of the pressure in the yielded regions which must be a function which can be extended continuously into unyielded zones [33].

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