ΑΠΑΝΤΗΣΕΙΣ ΤΩΝ ΑΣΚΗΣΕΩΝ

Κεφάλαιο 1

2 (i) $\frac{41}{333}$ (ii) $\frac{115}{9}$ (iii) $\frac{20943}{550}$ (iv) $\frac{537}{1250}$

3 (i) $x \in (-\infty, -2] \cup (2, +\infty)$ (ii) $x \in (-\infty, -2) \cup (4, +\infty)$
(iii) $x \in (-\infty, -\frac{5}{2}] \cup (-1, 2)$ (iv) $x \leq -2$ (Η ισότητα ισχύει και για $x = 1$.)

5 (i) Αδύνατη (ii) $x = 1$ ή $x = -\frac{7}{3}$ (iii) $x = -2$ ή $x = 10$ (iv) $x = \frac{17}{4}$ ή $x = \frac{23}{6}$
(v) $x = -\frac{1}{6}$ ή $x = 2$

6 (i) $x \leq -\frac{11}{2}$ (ii) $x \in (-\infty, -7) \cup (-7, -\frac{3}{2})$ (iii) $x < 0$ ή $0 < x < \frac{1}{4}$ (iv) $x \leq -4$ ή $x \geq -1$
(v) $x > 5$ ή $x < \frac{3}{5}$

7 (i) $M = 58$ (ii) $M = 18$

8 (i) $x \in \mathbb{R},\ f \in [-\frac{1}{4}, +\infty)$
(ii) $x \in \mathbb{R},\ f \in \mathbb{R}$
(iii) $x \in \mathbb{R} - \{0\},\ f \in \mathbb{R}$
(iv) $x \in \mathbb{R} - \{0\},\ f \in (-\infty, -2] \cup [2, +\infty)$
(v) $x \in [-1, 1],\ f \in [1, \frac{5}{4}]$
(vi) $x \in \mathbb{R},\ f \in [-2, +\infty)$
(vii) $x \in \mathbb{R} - \{-1, 1\},\ f \in (-\infty, -1] \cup [0, +\infty)$

10 $x \in \mathbb{R} - \{1, 5\}$. Για το δεύτερο ερώτημα δεν υπάρχει μοναδική λύση. Μια λύση είναι η: $f_1(x) = \frac{1}{x},\ f_2(x) = x^2 + x - 6,\ f_3(x) = |x - 3|$.

11 $c = 5$

12 (i) $\frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x}) + \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$
(ii) $\frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x}) \sin x + \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x}) \sin x$
(iii) $\frac{1}{2}\left[(10^x + 2x) + (10^{-x} - 2x)\right] + \frac{1}{2}\left[(10^x + 2x) - (10^{-x} - 2x)\right]$
(iv) $\frac{1}{2}\left[\sqrt{\frac{x-1}{x+1}} + \sqrt{\frac{x+1}{x-1}}\right] + \frac{1}{2}\left[\sqrt{\frac{x-1}{x+1}} - \sqrt{\frac{x+1}{x-1}}\right]$

13 (i) $\sqrt{1 - x^2} + \sin 3x,\ x \in [-1, 1]$
(ii) $\sqrt{1 - x^2} - \sin 3x,\ x \in [-1, 1]$
(iii) $\sqrt{1 - x^2} \cdot \sin 3x,\ x \in [-1, 1]$
(iv) $\frac{\sqrt{1-x^2}}{\sin 3x},\ x \in [-1, 0) \cup (0, 1]$
(v) $\sqrt{1 - \sin^2 3x},\ x \in \mathbb{R}$
(vi) $\sin\left(3\sqrt{1 - x^2}\right),\ x \in [-1, 1]$

14 (i) Όχι (ii) Ναι (iii) Ναι (iv) Ναι

17 (i) $x^{1/4} - 2,\ \ x \in [16, +\infty)$
(ii) $x^2 - 3,\ \ x \in [0, +\infty)$
(iii) $\frac{1}{6}(-5 + \sqrt{12x + 49}),\ \ x \in \left[-\frac{49}{12}, +\infty\right)$
(iv) $\frac{1}{10}(1 + \sqrt{1 - 20x}),\ \ x \in \left(-\infty, \frac{1}{20}\right]$

18 $x = \frac{8}{5}$

19 $c = -2$

20 (i) $y \neq 4$ (ii) $y \leq 5$ (iii) $y \in \{-1, 1\}$ (iv) $y \leq 0$

21 (i) $(-\infty, -4] \cup (0, +\infty)$ (ii) $[1, +\infty)$ (iii) $[-1, 2]$

23 $-5 \leq x \leq 1,\ 0 \leq y \leq 3$

24 $y = \frac{1}{x}\sqrt{\frac{x-1}{x+1}}\mathrm{e}^{-3x-c}$

26 (i) 12 (ii) $\sqrt{5}$ (iii) 4 (iv) -1 (v) $\frac{1}{4}$ (vi) $-\frac{1}{2}$ (vii) 0 (viii) $\frac{5}{2}$ (ix) $\frac{1}{3}$ (x) k^2 (xi) $3-k$ (xii) 0 (xiii) 1 (xiv) 1 (xv) 2 (xvi) 1 (xvii) $\frac{1}{3}$ (xviii) **0** (xix) **2** (xx) **1** (xxi) -1 (xxii) Δεν υπάρχει (xxiii) $-\frac{1}{10}$ (xxiv) $-\frac{1}{2}\sqrt{2}$

27 (i) Συνεχής στο $\mathbb{R}-\{1\}$ (ii) Συνεχής στο $\mathbb{R}-\{-2\}$
(iii) Συνεχής στο $[-1,4)\cup(4,+\infty)$ (iv) Συνεχής στο $\mathbb{R}-\{-1\}$

28 $k=3$

29 $a=-1,\ b=2$

30 $a=1,\ b=2$

31 $\lambda=\frac{\sqrt{2}-3}{6}$

32 Συνεχής στο $\mathbb{R}-\{0\}$

34 (i) $x=-1,1$ διορθώσιμα σημεία (ii) $x=-5,3$ διορθώσιμα σημεία

35 $k=\frac{1}{8}$

38 (i) $\frac{5}{4}$ (ii) $-\frac{3}{4}$ (iii) $\frac{34}{16}$

39 (i) $\frac{13}{12}$ (ii) $-\frac{12}{5}$ (iii) $\frac{13}{5}$ (iv) $-\frac{312}{25}$

40 $x=\frac{6}{5}$

41 $x=\log 2,\ \log 3$

42 $\alpha=\log 2,\ R=4,\ x=\log 6$

43 $+\infty$ αν $a>1$, $\frac{1}{2}$ αν $a=1$, 0 αν $0<a<1$

46 $\frac{4}{3},\ 5$

47 (i) $-\frac{1}{4}$ (ii) 0 (iii) $\frac{\sqrt{2}}{8}$ (iv) 0 (v) $\frac{1}{2}$ (vi) 0 (vii) $\frac{1}{2}m(m+1)$ (viii) 100 (ix) $\frac{2\sin a}{\cos^3 a}$

Κεφάλαιο 2

1 (i) $-\frac{1}{(x+1)^2}$ (ii) $\frac{1}{2\sqrt{x+1}}$ (iii) $\frac{1}{3x^{2/3}}$ (iv) $-\sin x$ (v) $\sec^2 x$ (vi) $\tan x\sec x$

2 $a=3,\ b=2$

6 $4f(x)f'(x)$

7 (i) $k\in\mathbb{R}$ (ii) $k=2$

10 (i) $t=-\frac{1}{2}$ (ii) $t=1,\ t=4$

11 $y=5x-14,\ \ y=6x-17$

13 (i) $-\frac{1}{x^2+1}$ (ii) $\frac{e^{2x}+2xe^{2x}}{1+x^2e^{4x}}$ (iii) $\frac{2x\log x+x}{\sqrt{1-x^4\log^2 x}}$ (iv) $\frac{x\log x-(1+x^2)\tan^{-1}x}{x(1+x^2)\log^2 x}$ (v) $\frac{x(8-5x)}{\sqrt{2-x}}$ (vi) $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
(vii) $2x(x^2+3)^3(2x^3-5)^2(17x^3+27x-20)$ (viii) $\frac{36x^2(x^3-1)^3}{(2x^3+1)^5}$

14 (i) $\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$ (ii) $6x^2(x+2)^2(x+1)$

15 (i) $\frac{(-1)^n(n+1)!}{x^{n+2}}$ (ii) $\frac{(-1)^n 3^n n!}{(3x+2)^{n+1}}$

16 (i) $2\sqrt{x-5}$ (ii) δεν υπάρχει αντίστροφη συνάρτηση (iii) $\frac{1}{5(x+2)^2}$

17 (i) $y''=\frac{2(1+y)}{(1+x)^2}$ (ii) $y''=-\frac{4xy}{(y^2-x)^3}$

21 (i) $y'=-2\sin t,\ y''=-1$ (ii) $y'=-\tan\theta,\ y''=\frac{1}{3\cos^4\theta\sin\theta}$

22 (i) $5x+3y-30=0,\ 3x-5y+16=0$ (ii) $2x+2y-a=0,\ x-y=0$

23 $\frac{3}{(x+1)^2}\sin\left[\left(\frac{2x-1}{x+1}\right)^2\right]$

24 (i) $\frac{2\tanh^{-1}x}{1-x^2}$ (ii) $\frac{1}{\sqrt{x^2-1}\cosh^{-1}x}$ (iii) $\frac{1}{\sqrt{(1+x^2)([\sinh^{-1}x]^2-1)}}$

26 $\frac{a^{2/3}}{3\sqrt{a^{2/3}-1}}$

27 e^{-2}

28 $f(x) = x^3$

29 $ay + bx = 2ab$

30 (i) $f_1(x) = \frac{(-1)^n n!}{4a}(x + na),\ f_2(x) = \frac{(-1)^{n+1} n!}{4a}(x - na)$
(ii) $\frac{(-1)^n n! n}{4a^{n+2}}[1 + (-1)^{n+2}]$

31 (i) $f^{(n)} = \frac{3(-1)^n n!}{(x-1)^{n+1}} + \frac{(-1)^n n!}{(x+1)^{n+1}} - \frac{8n! 2^n (-1)^n}{(2x-1)^{n+1}}$ (ii) 3024

32 (i) $g(x) = \frac{1}{x^5}\mathrm{e}^{\frac{1}{x}}$ (ii) $\frac{1}{32}\sqrt{\mathrm{e}}$

33 (i) $f(x) = x^2 + 6$ (ii) $g(n) = n^2 - 5n + 6$

34 $g(x, y) = \mathrm{e}^y - \mathrm{e}^x$

35 $f(n) = -n^2$

36 (i) $f(y) = -\frac{b^4}{a^2 y^3}$ (ii) $g(y) = -\frac{3b^6}{a^4 y^5}$

37 $\delta = -b^{2m}$

38 $(-1)^n a^{2m}(x^2 + 1)\sin ax + 4(-1)^m a^{2m-1} x \cos ax + (-1)^{m-1} 2m(2m-1)a^{2m-2}\sin ax$

39 (i) $\theta = \tan^{-1}\frac{b}{a}$ (ii) $2^{\frac{n}{2}}\cos(\frac{n\pi}{a})$

40 (i) $a_n = \frac{n}{4}$ (ii) $F^{(n)}(0) = 0$ για n άρτιο, $F^{(n)}(0) = 2(\sqrt{2})^n \sin\frac{n\pi}{4}$ για n περιττό

41 (i) $f(x) = 1 - x^2$ (ii) $y^{(2n+2)}(0) = 0,\ y^{(2n+3)}(0) = (2n+1)^2(2n-1)^2 \ldots 3^1 \cdot 1^2$

42 (i) $f(x) = x$ (ii) $g(n) = -(n-1)^2$

43 (i) $f(m) = m^2$ (ii) $g(n, m) = m^2 - n^2$

Κεφάλαιο 3

1 (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{5}{12}$ (iv) $\frac{1}{3}$ (v) $-\frac{2}{3}$ (vi) $\frac{2}{9}$ (vii) 2 (viii) 2 (ix) e^{-6} (x) e^{-2} (xi) e (xii) $\frac{1}{2}$ (xiii) $-\log 2$ (xiv) 4 (xv) $\log 3$ (xvi) 1 (xvii) e^a (xviii) 1 (xix) e^3 (xx) $-\frac{1}{\pi}$ (xxi) $-\frac{1}{6}$ (xxii) $-\frac{1}{2}$ (xxiii) 0 (xxiv) 1 (xxv) $\frac{1}{\mathrm{e}}$ (xxvi) $\frac{1}{3}$

2 $a = -1,\ \ b = \pm 2\sqrt{2}$

3 $\frac{at^2}{2}$

7 $c = \frac{-1+\sqrt{13}}{3}$

13 $c = -\frac{\pi}{2}$

15 $a = 5,\ \ b = \frac{33}{4}$

16 (i) φθίνουσα: $(-\infty, -2]$ και $[0, 2]$, αύξουσα: $[-2, 0]$ και $[2, +\infty)$
(ii) φθίνουσα: $[-2 - \sqrt{7}, -2 + \sqrt{7}]$, αύξουσα: $(-\infty, -2 - \sqrt{7}]$ και $[-2 + \sqrt{7}, +\infty)$
(iii) φθίνουσα $\forall x \in \mathbb{R}$

18 (i) Α.Ε.$= \sqrt{3}$ Α.Μ.$= 2$
(ii) Α.Ε.$= 0$ Α.Μ.$=18$
(iii) Α.Ε.$= -\sin 1$ Α.Μ.$= \sin 1$
(iv) Δεν υπάρχουν απόλυτα ακρότατα

19 (i) Α.Ε.$= -\frac{1}{4}$ Α.Μ.$= 2$

21 Αν $p > 0$, min $= q - 2p^{\frac{3}{2}}$, max $= q + 2p^{\frac{3}{2}}$. Αν $p \leq 0$ δεν υπάρχουν.

22 min στο (-1,-3), max στο (5,3)

23 $y = 1$, Τ.Ε.(Α.Ε.) $(1, 0)$, Τ.Μ.(Α.Μ.) $(-1, 4)$

24 $a = 1, \;\; b = -3$

25 (i) $0, 20$ (ii) $8, 12$

26 $(\pm\sqrt{2}, 1)$

27 $(-1, 0)$

28 $2\sqrt{2}$

29 $(\frac{1}{2}, -\frac{1}{2})$ και $(-\frac{1}{2}, \frac{1}{2})$

30 (i) $(2, \pm\sqrt{3})$ (ii) $(1, 0)$ (iii) $(1, 0)$

33 (i) $x + \frac{1}{3}x^3$ (ii) $x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$
(iii) $1 + \frac{1}{2}x^2 + \frac{5}{24}x^4$ (iv) $\log 3 + \frac{2}{3}x - \frac{2}{9}x^2 + \frac{8}{81}x^3 - \frac{4}{81}x^4$

34 (i) $\displaystyle\sum_{k=0}^{\infty}(-1)^k x^k$ (ii) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}$
(iii) $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}(2k)!}x^{2k}$ (iv) $\displaystyle\sum_{k=0}^{\infty}\frac{1}{(2k)!}x^{2k}$

35 (i) $\displaystyle-\sum_{k=0}^{\infty}(x+1)^k$ (ii) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(x-1)^k$
(iii) $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k\pi^{2k}}{(2k)!}(x-\tfrac{1}{2})^{2k}$ (iv) $\displaystyle\sum_{k=0}^{\infty}\frac{16-(-1)^k}{8k!}(x-\log 4)^k$

36 (i) $R_3(x) = \frac{(4+c)\mathrm{e}^c}{4!}x^4$ (ii) $R_2(x) = \frac{3c^2-1}{3(1+c^2)^3}x^3$
(iii) $R_4(x) = \frac{\cos c}{5!}(x - \frac{\pi}{6})^5$ (iv) $R_5(x) = \frac{7}{(1+c)^8}(x+2)^6$

39 (i) $x - x^2 + \frac{x^3}{2!} - \frac{x^4}{3!} + \ldots, \;\; (-\infty, +\infty)$
(ii) $x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} - \frac{x^8}{6!} + \ldots, \;\; (-\infty, +\infty)$
(iii) $x^2 - \frac{2^3}{4!}x^4 + \frac{2^5}{6!}x^6 - \frac{2^7}{8!}x^8 + \ldots, \;\; (-\infty, +\infty)$
(iv) $-x^2 - \frac{1}{4}x^4 - \frac{1}{3}x^6 - \frac{1}{4}x^8 + \ldots, \;\; (-1, 1)$
(v) $x^2 - 3x^3 + 9x^4 - 27x^5 + \ldots, \;\; (-\frac{1}{2}, \frac{1}{3})$
(vi) $x + \frac{1}{2}x^3 + \frac{3}{5}x^5 + \frac{5}{16}x^7 + \ldots, \;\; (-1, 1)$

40 $\displaystyle\sum_{k=0}^{\infty}(-1)^k(x-1)^k, \;\; (0, 2)$

41 (i) $\sin\pi = 0$ (ii) $\mathrm{e}^{-\log 3} = \frac{1}{3}$

42 $-\frac{1}{2}$

43 (i) $\frac{40\sqrt{3}}{9+4\sqrt{3}}$ (ii) 10

44 $\frac{\sqrt{3}}{2}R$

45 (i) $\frac{4\pi r^3}{3\sqrt{3}}$ (ii) $\pi r^2(1+\sqrt{5})$

46 $(\frac{\sqrt{2}}{2}a, a)$

47 $\frac{4}{3}R$

48 $\frac{4}{3}R$

49 $(a - 2p, \pm\sqrt{4p(a-2p)})$

50 $\frac{\sqrt{5}(1-\cos\theta)}{\sqrt{\sin\theta}}, \;\; \frac{\sqrt{5}}{\sqrt{\sin\theta}}(1+\cos\theta), \;\; \frac{\sqrt{5}}{\sqrt{\sin\theta}}$

Κεφάλαιο 4

1 (i) $\frac{2}{7}x^{7/2}+\frac{8}{9}x^{9/2}+c$ (ii) $\frac{2}{3}x^{3/2}-2x^{1/2}+c$ (iii) $\sec x+c$ (iv) $\frac{\pi^2}{9}+2\sqrt{3}$ (v) $\sqrt{2}-1$ (vi) 20

2 $6x+\frac{5}{2}\cos 2x+\frac{1}{2}$

3 $\frac{2}{3}x^{3/2}+\frac{13}{3}$

4 (i) 20 (ii) $\frac{17}{2}$ (iii) $\frac{9}{4}\pi+6$ (iv) π (v) $\frac{25}{2}\pi$ (vi) $\frac{3}{2}\pi$ (vii) 48

7 (i) $\frac{2}{3}$ (ii) $\frac{1}{5}$

8 (i) $x=\frac{25}{4}$ (ii) $x=\frac{1}{3}$ (iii) $x=3$

9 (i) $x^*=-\frac{5}{4}$ (ii) $x^*=\frac{\pi}{4},\ \frac{3\pi}{4}$ (iii) $x^*=\sqrt{3}$

11 (i) $3x^2\sin^2(x^3)-2x\sin^2(x^2)$
(ii) $\frac{2}{1-x^2}$
(iii) $\left(2x+\frac{1}{2\sqrt{x}}\right)\left(x^2+\sqrt{x}+\sqrt{x^2+\sqrt{x}}\right)$

13 $A=3,\ \ B=18$

14 (i) $F(1)=0$ (ii) $F'(1)=\frac{1}{2}$

17 (i) 1 (ii) $\frac{1}{4}$ (iii) $\sqrt[3]{12}$

18 $k=4$

19 (i) $2\sqrt{x}\log x-4\sqrt{x}+c$
(ii) $\frac{1}{2}(x^2\tan^{-1}x-x+\tan^{-1}x]+c$
(iii) $-\frac{1}{6}\mathrm{e}^{-3x}(\sin 3x+\cos 3x)+c$
(iv) $\frac{1}{2}x(\cos(\log x)+\sin(\log x))+c$
(v) $x\sin^{-1}x+\sqrt{1-x^2}+c$
(vi) $\frac{\mathrm{e}^x}{x+1}+c$
(vii) $-\frac{1}{x}(\log x+1)+c$
(viii) $\frac{1}{2}\log(x^2+6x+13)-\frac{3}{2}\tan^{-1}(\frac{x+3}{2})+c$

23 (i) $\sec x+c$
(ii) $-\frac{2}{3}(\cos x)^{2/3}+c$
(iii) $\frac{1}{7}\sec^7 x-\frac{1}{5}\sec^5 x+c$
(iv) $-\cot x-\frac{1}{3}\cot^3 x+c$ (v) $\tan^{-1}(\mathrm{e}^x+1)+c$ (vi) $\frac{1}{2}\left(\tan^{-1}(x+2)\right)^2+c$ (vii) $\frac{1}{5}\cosh^5 x-\frac{1}{3}\cosh^3 x+c$ (viii) $\sinh^{-1}\frac{x-1}{4}+c$ (ix) $-\frac{1}{4}\coth^{-1}\left(x+\frac{3}{2}\right)+c$ (x) $x+\ln x+c$ (xi) $-\frac{1}{4}\ln|2x+1|+\frac{x}{2}+c$ (xii) $\tan x+\mathrm{e}^{\sin x}+c$

24 $-\frac{1}{\sqrt{2}}\log\left|\mathrm{cosec}\left(x+\frac{\pi}{4}\right)+\cot\left(x+\frac{\pi}{4}\right)\right|+c$
$-\frac{1}{\sqrt{a^2+b^2}}\log\left|\mathrm{cosec}\left(x+\tan^{-1}\frac{b}{a}\right)+\cot\left(x+\tan^{-1}\frac{b}{a}\right)\right|+c$

25 (i) $-\frac{\sqrt{25+x^2}}{25x}+c$ (ii) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)+c$
(iii) $2-\frac{\pi}{2}$ (iv) $\frac{5}{24\sqrt{3}}$

26 $\frac{1}{2}\log(x^2+4)+c$

27 (i) $\frac{1}{4}\tan^{-1}(4x+2)+c$
(ii) $\sinh^{-1}\left(\frac{2\mathrm{e}^x+1}{\sqrt{3}}\right)+c$
(iii) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sin x-3}{\sqrt{3}}\right)+c$
(iv) $\frac{1}{4}\log(4x^2+4x+5)+\frac{1}{2}\tan^{-1}\left(x+\frac{1}{2}\right)+c$
(v) $\log x-\log\sqrt{x+2+1}+c$
(vi) $\frac{1}{3}x^3+\frac{1}{2}\log(x^2+6x+10)-3\tan^{-1}(x+3)+c$

(vii) $-\log(\mathrm{e}^{-x}+1)+c$
(viii) $\frac{1}{\tan x}-\log|\tan x|+\log|\tan x-1|+c$

28 $a=\sqrt{2},\ \ b=-\sqrt{2}$

29 (i) $-x-4\sqrt{x}-4\log(\sqrt{x}-1)+c$
(ii) $2\sqrt{x}\mathrm{e}^{\sqrt{x}}-2\mathrm{e}^{\sqrt{x}}+c$
(iii) $\frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{2\tan(x/2)+1}{\sqrt{3}}\right)+c$
(iv) $-x+\frac{4}{\sqrt{3}}\tan^{-1}(\sqrt{3}\tan\frac{x}{2})+c$

30 (i) $-\frac{(4-x^2)^{3/2}}{12x^3}+c$
(ii) $-\frac{\sqrt{3-x^2}}{3x}+c$
(iii) $-\frac{\sqrt{x^2+1}}{x}+c$
(iv) $\frac{1}{15}\left(1-\frac{5}{x^2}\right)^{3/2}+c$

31 (i) $2\sqrt{x}+\frac{2}{3}x^{3/2}+c$
(ii) $-\frac{\sqrt{1-x^2}}{x}-\sin^{-1}x+c$
(iii) $2\sqrt{x}-4x^{\frac{1}{4}}+\log(x^{1/4}+1)+c$
(iv) $-\frac{1}{4}\log(\cos x+1)+\frac{1}{4}\log(\cos x-1)-\frac{1}{2(\cos x-1)}+c$

33 $-\frac{\pi^2}{2}$

34 (i) $-\frac{1}{3}\left(\frac{2}{x-1}+1\right)^{3/2}+c$ (ii) $(x-1)\sqrt{\frac{1+x}{1-x}}+2\tan^{-1}\sqrt{\frac{1+x}{1-x}}+c$ (iii) $\frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{x-1}+\sqrt{3}}{\sqrt{x-1}-\sqrt{3}}\right|+c$

36 (i) $\frac{2}{3}\log 2-\frac{5}{18}$ (ii) 1

39 (i) $\ln(3x^2+6x+28)+c$ (ii) $\frac{\sqrt{3}}{15}\tan^{-1}\left(\frac{3x+3}{5\sqrt{3}}\right)+c$
(iii) $\frac{1}{3}\ln(3x^2+6x+28)-\frac{\sqrt{3}}{15}\tan^{-1}\left(\frac{3x+3}{5\sqrt{3}}\right)+c$

40 (i) $\frac{1}{3}\mathrm{e}^{-x}+\frac{1}{9}\ln\left|\frac{\mathrm{e}^x-3}{\mathrm{e}^x}\right|+c$ (ii) $\ln\left|\frac{\sqrt{1+\cos^2 x}}{\cos x}\right|+c$ (iii) $\ln|1+\tan x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+c$

41 (i) $\frac{\sqrt{3}}{3}\ln\left|\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right|+c$ (ii) $\frac{2}{\sqrt{3}}\tan^{-1}(\sqrt{3}\tan\frac{x}{2})+c$ (iii) $\ln|1+\tan\frac{x}{2}|+c$

42 (i) $-\sec(\sin^{-1}\frac{1}{x})+c$ (ii) $-\frac{3}{4}(1-x)^{\frac{4}{3}}+\frac{9}{7}(1-x)^{\frac{7}{3}}-\frac{9}{10}(1-x)^{\frac{10}{3}}+\frac{3}{13}(1-x)^{\frac{13}{3}}+c$
(iii) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+c$

43 (i) $2(\tan\frac{x}{2}+\sec\frac{x}{2})+c$ (ii) $\frac{1-\cos 3x}{3\sin 3x}+c$ (iii) $\frac{1}{3}\sin^{-1}\ln x^{\frac{3}{2}}+c$ (iv) $\frac{2}{3}\ln(\mathrm{e}^{2x}+3)-\frac{1}{3}x+c$
(v) $\frac{3}{2}\ln[c(x^{\frac{2}{3}}+1)],\ c>0$ (vi) $x+\frac{1}{2}(\cot 2x-\csc 2x)+c$

44 (i) $\frac{1}{2}$ (ii) Δεν συγκλίνει (iii) $\frac{\pi}{2}$ (iv) 6 (v) $-\sqrt{8}$ (vi) $3\sqrt[3]{2}$ (vii) Δεν συγκλίνει (viii) $\frac{\pi}{2}$ (ix) $\frac{\pi}{2}$ (x) -1 (xi) 0 (xii) $\frac{1}{9}$ (xiii) Δεν συγκλίνει (xiv) $-\frac{4}{9}$ (xv) $2(1-\mathrm{e}^{-2})$ (xvi) 2 (xvii) $\frac{\pi}{4}$ (xviii) $-\frac{1}{4}$ (xix) $\frac{1}{4}$ (xx) $\frac{1}{\ln 2}$ (xxi) 6

45 (i) $a=\frac{1}{5}$ (ii) $a=\frac{\pi}{2}$

46 (i) $\sqrt{\pi}$ (ii) $\frac{\sqrt{\pi}}{2a}$

47 $\frac{8\sqrt{2}}{5}$

50 (i) $\ln\left|\frac{u+\sqrt{u^2+a^2}}{a}\right|$ (ii) $\frac{1}{\sqrt{2}}\ln(1+\sqrt{2})$ (iii) $\frac{16}{3}\pi-2\sqrt{3}$ (iv) $\frac{1}{6}$ (v) $\frac{1}{a^2-b^2}\ln\left|\frac{a}{b}\right|$ (vi) $\frac{\pi}{3\sqrt{3}}$ (vii) $\frac{1}{16}\pi^4-3\pi^2+24$ (viii) $\ln\frac{3}{2}$ (ix) $\ln\frac{4}{3}$ (x) $\frac{a}{a^2+b^2}$ (xi) 1

51 $\frac{q!p!}{(p+q+1)!}$

52 $x=2$

53 (ii) $\frac{\pi}{2^{n+1}}$

54 (i) $a_n = \frac{2n}{2n+1}$ (ii) $b_n = \frac{2^{2n+1}}{(2n+1)!}$

55 (i) $I_1 = \frac{\pi}{4}$ (iii) $I_2 = \frac{\pi}{2\sqrt{2}}$

Κεφάλαιο 5

1 (i) $\frac{11}{2}$ (ii) $\frac{9}{2}$ (iii) $4\sqrt{2}$ (iv) 24 (v) $\frac{9}{2}$ (vi) $\frac{98}{3}$

2 $k = \sqrt[3]{4}$

4 (i) $\frac{\pi(2-\sqrt{2})}{2}$ (ii) $\frac{2048}{15}\pi$ (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi}{6}$ (v) $\frac{16}{15}\pi$ (vi) $\frac{81}{10}\pi$ (vii) $\frac{1}{2}(\mathrm{e}^4-1)\pi$ (viii) $\frac{3}{5}\pi$ (ix) 5π

5 (i) $\frac{8}{3}\pi$ (ii) $\frac{3}{10}\pi$ (iii) $\frac{16}{3}\pi$ (iv) $\frac{\pi}{3}$ (v) $128\sqrt{3}\pi$ (vi) $\pi(1-\frac{1}{e})$

6 (i) $\frac{256}{5}\pi$ (ii) $\frac{512}{15}\pi$ (iii) 8π (iv) $\frac{1088}{15}\pi$ (v) $\frac{128}{3}\pi$

7 $\frac{7}{4}\pi$

8 $\frac{9}{14}\pi$

9 (i) 12 (ii) $\frac{14}{3}$ (iii) $\frac{53}{6}$ (iv) $\frac{2(10\sqrt{10}-1)}{27}$ (v) $2-\frac{\pi}{4}$ (vi) $\ln\frac{21}{5}-\frac{1}{2}$ (vii) 14 (viii) $\frac{17}{12}$ (ix) $\frac{1}{2}e^2-\frac{1}{4}$ (x) $\ln(1+\sqrt{2})$ (xi) 16

10 (i) $\frac{17\sqrt{17}-5\sqrt{5}}{6}\pi$ (ii) $\frac{515}{64}\pi$ (iii) $\frac{\pi}{27}\left[10\sqrt{10}-1\right]$ (iv) π (v) $\frac{2\sqrt{2}\pi(2\mathrm{e}^{\pi}+1)}{5}$

11 (i) $\frac{1823}{18}\pi$ (ii) $\frac{738}{5}\pi$

12 $\sqrt{5}+\sqrt{2}+\log(1+\sqrt{2})$, $\frac{8}{3}\pi\left(2^{\frac{3}{2}}-1\right)+2\pi\sqrt{5}=\frac{16}{3}\pi\sqrt{2}-\frac{8}{3}\pi+2\pi\sqrt{5}$

13 $V=(\tan^{-1}b-\frac{\pi}{4})\pi$, $\lim\limits_{b\to+\infty}V=\dfrac{\pi^2}{4}$

14 $2\left(1-\frac{1}{1+\sqrt{3}}\right)\pi$

15 (i) $\frac{2048\pi}{5}$ (ii) $\frac{32\pi}{3}$

16 $\frac{64\pi}{3}$

17 20

18 $\ln(2+\sqrt{3})$

19 3π

20 2

21 (i) 4π (ii) $\frac{1}{4}$ (iii) 2

22 $6\mathrm{e}^{-1}-1$

23 (i) $\frac{1}{3}\pi r^2h$ (ii) $\pi r\sqrt{r^2+h^2}$ (iii) $h=\sqrt{3}R$

24 (ii) 3π

25 (i) ΜΙΝ $f(0)=0$ ΜΑΞ $f(\frac{1}{2})=1$ (ii) $\frac{1}{2}\pi\mathrm{e}$

26 (ii) $\frac{\sqrt{\pi}}{2}$

27 (i) $\frac{1}{4}\pi(4-\pi)$ (ii) $\pi(\sqrt{5}-\sqrt{2}+\ln\left(\frac{2(\sqrt{2}+1)}{\sqrt{5}+1}\right)$

28 $E_1=2\pi+\frac{4}{3}$, $E_2=6\pi-\frac{4}{3}$

29 $\frac{32}{27}$

30 (i) $4\pi^2$ (ii) $2\pi^2$

31 (i) $32\pi^2$ (ii) $32\pi^2$

32 $\frac{6}{5}\pi a^2$

33 $2\pi[\sqrt{2}+\ln(\sqrt{2}+1)]$
34 (i) $\frac{512}{15}\pi$ (ii) $\frac{1}{6}\pi(17^{3/2}-1)+4\pi$
35 $\sqrt{2}a^2\pi(2-\frac{1}{2}\pi)$
36 $2\pi^2R(ab-cd)$

Κεφάλαιο 6

2 (i) $x^2y''-4xy'+6y=0$ (ii) $xy'+2y=0$
3 (i) $y=cx$ (ii) $y=c\mathrm{e}^{-\sqrt{1+x^2}}-1$ (iii) $y=1-c\mathrm{e}^{-x-\frac{1}{3}x^3}$ (iv) $y=\ln(\sec x+c)$
4 (i) $\ln|x|-\frac{1}{3}\left(\frac{y}{x}\right)^3=c$ (ii) $y+\sqrt{x^2+y^2}=c$ (iii) $y^2-4xy+x^2=c$ (iv) $y^3=x^3(c-6\ln x)$
5 (i) $y=\mathrm{e}^{-2x}+c\mathrm{e}^{-3x}$ (ii) $y=\mathrm{e}^{-x}\sin(\mathrm{e}^x)+c\mathrm{e}^{-x}$ (iii) $y=\frac{1}{2}\sinh x+\frac{1}{2}x\mathrm{sech}x+c\mathrm{sech}x$ (iv) $y=cx+x\sin x$
6 (i) $y=-\ln(\mathrm{e}^{-2}-\frac{1}{2}x^2)$ (ii) $y=x\cos x-\sin x\cos^2 x+\cos x$
8 (i) $y=\left(\frac{c}{x^2}-\frac{2}{3}x\right)^{-\frac{1}{2}}$ (ii) $y=\left(c\mathrm{e}^{\frac{x^2}{2}}-1\right)^2$
9 $a=\pm1,\ b=1,\ y=\mathrm{e}^x+\frac{2}{2c\mathrm{e}^x-\mathrm{e}^{-x}}$
10 $y=(c_1x+c_2)^2$
11 $y=\tan^{-1}(\frac{x^2}{3}+\frac{c}{x})$
12 (i) $y=x$ (ii) $x=y\mathrm{e}^{1-y}$ (iii) $y=-\cos\frac{x}{2}+\frac{2}{x}\sin\frac{x}{2},\ \ 0$ (iv) $y=\frac{1}{4}x^4\ln^2 x$ (v) $y=\frac{2x-1}{2x+1}$ (vi) $x=\frac{y+1}{\ln(1+y)+\frac{1}{2}}$ (vii) $y=x^2+1$ (viii) $y=\left(\tan x+\frac{\ln|\cos x|+1}{x}\right)^2$ (ix) $y=x^2(1+\mathrm{e}^{\frac{1}{x}-1})$ (x) $y+x=a\tan(\frac{y}{a}+\frac{\pi}{4})$ (xi) $y=ax+x\sqrt{1-x^2}$ (xii) $y=\frac{4}{2\ln x+15x^2+1}$ (xiii) $y=(\ln x+\frac{1}{2}x^2-\frac{1}{2})\mathrm{e}^x$ (xiv) $y\mathrm{e}^{x/y}+x=2\sqrt{\mathrm{e}}+1$ (xv) $y=-x\tan^{-1}x$ (xvi) $y=\frac{1}{9}(x^2-1)^2$ ή $y=[\frac{1}{3}(x^2-1)-\frac{2}{3}(x^2-1)^{1/4}]^2$ (xvii) $y=\frac{1}{2}\sqrt{\frac{1+x}{1-x}}(\sin^{-1}x+x\sqrt{1-x^2}+2)$ (xviii) $\ln x=\mathrm{e}^{-2}-\mathrm{e}^{-\frac{x^2+y^2}{x}}$ (xix) $x^3=3\mathrm{e}^y-y-2$ (xx) $y^2-xy+x^2+x-y=2$ (xxi) $y=\frac{(1-b^2)x}{bx+1}+b$ (xxii) $y=\sqrt{10(x+1)-(1+x)\ln(1+x)-1}$ (xxiii) $y=\frac{2}{\cos x+\sin x}$

Κεφάλαιο 7

1 (i) $21+\mathrm{i}$ (ii) $-\frac{15}{2}+5\mathrm{i}$ (iii) $-\frac{11}{2}-\frac{23}{2}\mathrm{i}$ (iv) $-3-3\mathrm{i}$
2 (i) $-7+3\sqrt{3}+\sqrt{3}\mathrm{i}$ (ii) $765+128\sqrt{3}$ (iii) -35 (iv) $\frac{6\sqrt{3}+4}{7}$
3 $x=1,\ y=-2$
4 (i) $\sqrt{5}\mathrm{e}^{\mathrm{i}\tan^{-1}(1/2)}$ (ii) $5\mathrm{e}^{\mathrm{i}[\pi+\tan^{-1}(4/3)]}$ (iii) $\sqrt{5}\mathrm{e}^{-\mathrm{i}\tan^{-1}2}$
5 -1
6 $9,\ 6\theta$
7 $\frac{1}{2}(-1+i\sqrt{3}),\ -\frac{1}{2}(1+i\sqrt{3})$
8 (i) $\pm(1+i)$ (ii) $\pm\frac{\sqrt{3}-i}{\sqrt{2}}$
9 (i) $\pm\sqrt{2}(1+i),\ \pm\sqrt{2}(1-i)$ (ii) $\pm(\sqrt{3}-i),\ \pm(1+i\sqrt{3})$
11 2^{13}
14 6
17 $-\frac{1}{2}(1-i\sqrt{3}), -\frac{1}{2}(1+i\sqrt{3}), -2$
18 $y+x=0$

19 $\frac{1}{60}$

20 (i) $\frac{1}{2} + \frac{\sin[(n+\frac{1}{2})\phi]}{2\sin\frac{\phi}{2}}$ (ii) $\frac{1}{2}\cot\frac{\phi}{2} - \frac{\cos[(n+\frac{1}{2})\phi]}{2\sin\frac{\phi}{2}}$

21 (i) $1+i,\ -2-3i$ (ii) $2+5i,\ -3+2i$ (iii) $4+i,\ 1-3i$ (iv) $-2-i,\ 1-2i$ (v) $2-i,\ 1+2i$ (vi) $\pm\sqrt{3}\pm i,\ \pm 2i$ (vii) $\pm 1 + \pm 3i$ (viii) $\pm\frac{\sqrt{3}}{2} \pm \frac{1}{2}i,\ \pm\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ (ix) $\pm\frac{\sqrt{3}}{2} \pm \frac{1}{2}i,\ \pm\frac{1}{2} \pm i\frac{\sqrt{3}}{2},\ \pm 1$

22 (i) $\frac{\sqrt{3}}{2} - \frac{i}{2}$ (ii) $(-1)^n 2^{6n+1}$ (iii) 0 (iv) 0 (v) $1+i$

23 $c = 3$

24 $b = \alpha + \beta i,\ (x+\frac{\alpha}{4})^2 + (y+\frac{\beta}{4})^2 = \frac{|b|^2-4}{16}$