

ΑΠΑΝΤΗΣΕΙΣ ΤΩΝ ΑΣΚΗΣΕΩΝ

Κεφάλαιο 1

4. $a = -1, b = -2, c = 1$
5. (i) $y = \frac{2}{13}x^{\frac{13}{2}} + \frac{10}{3}x^{\frac{3}{2}} + c$ (ii) $y = -5e^{-x} + c$
 (iii) $y = -\frac{1}{2}x^4 + x^2 + c_1x + c_2$ (iv) $y = x^3 + x + \frac{c}{x}$.
6. (i) $c = 7$ (ii) $c = -\pi$ (iii) $c = \frac{\pi}{4}$ (iv) $c = -56$
7. (i) $\dot{\mathbf{r}} = (6 - 2e^{-t})\mathbf{i} + (5 \sin t - 3)\mathbf{j} + (3 \cos t - 1)\mathbf{k}$, $\mathbf{r} = (6t + 2e^{-t} - 1)\mathbf{i} + (2 - 5 \cos t - 3t)\mathbf{j} + (3 \sin t - t + 2)\mathbf{k}$
 (ii) $\dot{\mathbf{r}} = (4t^3 + 6)\mathbf{i} + (9t^2 - 8t + 15)\mathbf{j} - (3t^2 + 8)\mathbf{k}$, $\mathbf{r} = (t^4 + 6t + 3)\mathbf{i} + (3t^3 - 4t^2 + 15t - 1)\mathbf{j} + (4 - t^3 - 8t)\mathbf{k}$

Κεφάλαιο 2

1. (i) $(e^x + 1)^{-2} + 2(e^y + 1)^{-1} = c$ (ii) $(y + 1)^{-1} + \ln|y + 1| = \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + c$
 (iii) $\left(\frac{y+3}{x+4} \right)^5 = ce^{y-x}$ (iv) $y = \cot^{-1}(c - \cos x)$
 (v) $y = \sin \left(\frac{x^2}{2} + c \right)$ (vi) $y = \frac{1}{c - \tan^{-1}(e^x)}$
2. (i) $(x + 1)^6(y^2 + 1) = c(x + 2)^4$ (ii) $y^2 + xy = cx^3$
 (iii) $\sin \frac{y}{x} = cx$ (iv) $(x^2 + y^2)^{\frac{3}{2}} = x^3 \ln cx^3$
3. (i) $y(x) = -3\sqrt{1 + 2x}$ (ii) $y(x) = \sin^{-1}(x^2 + \frac{1}{2})$
 (iii) $4y^5 + 20y^2 = 5(2x - 1)e^{2x} + 21$ (iv) $y(x) = \tan \left(\frac{2x^2 - 12x + \pi}{4} \right)$
4. (i) $(x - y) \ln|x - y| = y + c(x - y)$ (ii) $x + y \ln|x| = cy$
 (iii) $\ln(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} = c$ (iv) $4x = y(\ln|y| - c)^2$
5. (i) $2 \frac{y}{x} + \ln \left| \frac{y}{2x^2} \right| = 4$ (ii) $y = \frac{x^3 + x}{2}$
 (iii) $y^3 + 3x^3 \ln|x| = 8x^3$ (iv) $\ln|x| = e^{\frac{y}{x}} - 1$
6. (i) $y = 1 + ce^{\frac{1}{x}}$ (ii) $3(x^2 + x)y = x^3 - 3x + c$
 (iii) $y = \frac{1 + ce^{-x}}{x}$ (iv) $y = (x + c) \cos x$
 (v) $2(1 + \sin x)y = x + \sin x \cos x + c$ (vi) $y = (x^3 + c)e^{-3x}$
7. (i) $y = x^4 - 2x^2$ (ii) $y = (e^x + 1)^2$
 (iii) $y = \frac{1}{2} \sin 2x + \frac{\sqrt{2}}{2} \cos x$ (iv) $y = \frac{1}{5}(2e^x - \sin 2x - 2 \cos 2x)$
8. (i) $y = \frac{x}{x+c}$ (ii) $y = \sqrt{2 + cx^{-1}e^{-x}}$
 (iii) $y^3 = 1 + cx^{-3}$ (iv) $y^{-3} = x + \frac{1}{3} + ce^{3x}$
9. (i) $x^2y^4 = x^4 + 15$ (ii) $\frac{1}{\sqrt{xy}} = -\frac{1}{2}x + 1$
 (iii) $y = \frac{1}{\sqrt{2e^{2x} - 1}}$ (iv) $y = -\frac{1}{4}e^{-2x}$
10. (i) $y = (x - 2 + ce^{-x})^{-1} + 1$ (ii) $\frac{e^{-\frac{x^2}{2}}}{y-x} = \int e^{-\frac{x^2}{2}} dx + c$
 (iii) $y = (2 + ce^{-2x^2})^{-1} + x$
11. (i) $y(x) = \frac{3x+2}{2x}$ (ii) $y(x) = \frac{x^3 - x - 1}{x^2 + x}$
 (iii) $y(x) = 2 + \frac{1}{e^x + \sin x}$
12. (i) $x^2y + x + 2y^2 = c$ (ii) $3x^2y + 2xy^2 - 5x - 6y = c$
 (iii) $(y^2 + 1) \sin x = c$ (iv) $y \tan x + \sec x + y^2 = c$

13. (i) $x^3y^2 - xy^3 + x^2 + y + 1 = 0$ (ii) $y^2 \cos x - y \sin^2 x = 9$
 (iii) $e^x y + xy^2 + 2e^x = 8$ (iv) $2x^{\frac{1}{3}}y^{-\frac{1}{3}} + 4x^{\frac{4}{3}}y^{\frac{1}{3}} = 9$
14. $\mu = x^2, x^4 + x^3y^2 = c$
15. (i) $\mu = |x^3|, 4x^5y + 4x^4y^2 + x^4 = c$ (ii) $\mu = |\cos x|, x^2 \cos y + x \sin y = c$
 (iii) $\mu =, xy^2e^x + ye^x = c$ (iv) $\mu = y^{-2}, x^2 + xy^{-1} + y^2 = c$
16. (i) $\mu = x^2y^3, x^3y^4(xy + 2) = c$ (ii)
17. (i) $5x^2 + 4xy + y^2 + 2x + 2y = c$ (ii) $x - 2y + \ln|3x - y - 2| = c$
 (iii) $\ln[c(x^2 + y^2 - 2x + 2y + 2)] + 4 \tan^{-1}(\frac{y+1}{y-1}) = 0$ (iv) $(2x + y + 3)^3(x - y + 1)^2 = c$
18. (i) $\ln[3(x-1)^2 + (y+3)^2] + \frac{2}{\sqrt{3}} \tan^{-1} \frac{y+3}{\sqrt{3}(x-1)} = \ln 4 + \frac{\pi}{3\sqrt{3}}$
 (ii) $x + 2y - 2 - \ln|2x + 3y - 1| = 0$
19. (i) $y = cx + \frac{1}{c^2}, 4y^3 = 27x^2$ (ii) $x = \frac{2p}{p^2-1}, y = \frac{2}{p^2-1} - \ln|p^2 - 1| + c$
 (iii) $y^2 = 2cx + c^3, 32x^3 + 27y^4 = 0, y = 0$ (iv) $2x = 1 + 2 \ln|y|, y^2 = 2cx - c \ln c$
 (v) $x = \frac{\ln p + c}{\sqrt{p}}, y = \sqrt{p}(4 - \ln p - c), y = 0$
20. $u' - Q(x)u = -P(x), y = e^{x^2+cx^{-2}}$
21. (i) $\sin^2 y = 4x^2 - cx$ (ii) $e^{2y} - x^2 + 2xe^y = c$
 (iii) $y = \left(6x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} + c\right)^{-1}$ (iv) $y = \frac{1}{3x+ct^2} + \frac{1}{t}$
 (v) $y^2 = c_1e^x + c_2$
22. (i) $\ln|x+y| + \frac{x}{x+y} = c, y = -x$ (ii) $8y - 4x + \ln|4x + 8y + 5| = c, 4x + 8y + 5 = 0$
 (iii) $ye^x(2x - y) = c$ (iv) $y = (1+x)(\frac{1}{2}x^2 + c)$
 (v) $x^2y^2 = -2x \cos x + 2 \sin x + c$ (vi) $4y = x^2, 5y = -5x^2 + 5cx - c^2$
23. (i) $x^2 + 3y^2 = k62$ (ii) $y^2(\ln y - \frac{1}{2}) = -x^2 + k$
 (iii) $x^2 + y^2 - 2x + 4 \ln(x+2) = k$ (iv) $x^2y = \frac{1}{4}x^4 + k$
 (v) $y = k(x^2 + 3y^2)$
24. (i) $\ln|2\sqrt{3}x^2 - xy + \sqrt{3}y^2| - \frac{6}{\sqrt{23}} \tan^{-1} \frac{2\sqrt{3}y-x}{\sqrt{23}x} = c$
 (ii) $\ln|3x^2 + 3xy + 4y^2| = -\frac{2}{\sqrt{39}} \tan^{-1} \left(\frac{3x+8y}{\sqrt{39}x}\right) = c$

Κεφάλαιο 3

1. (i) $W = e^{5x} \neq 0$ (ii) $y = c_1e^{2x} + c_2e^{3x}$ (iii) $y = 3e^{2x} - re^{3x}, x \in (-\infty, \infty)$
2. (i) $W = -\frac{4}{x} \neq 0$ (ii) $y = c_1x^2 + c_2x^{-2}$ (iii) $y = \frac{1}{4}x^2 + 8x^{-2}, x \in (0, \infty)$
3. (i) $y = c_1x + c_2(x-1)e^x$ (ii) $y = c_1e^{2x} + c_2(x+1)$ (iii) $y = c_1e^x + c_2x^{-1}$
4. (i) $y = c_1e^{\frac{x}{2}} + c_2e^{-2x}$ (ii) $y = (c_1x + c_2)e^{\frac{x}{2}}$
 (iii) $y = e^{-x}(c_1 \sin \frac{3}{4}x + c_2 \cos \frac{3}{4}x)$ (iv) $y = (c_1x + c_2)e^{4x}$
 (v) $y = e^{2x}(c_1 \sin 3x + c_2 \cos 3x)$ (vi) $y = c_1 \sin \frac{1}{2}x + c_2 \cos \frac{1}{2}x$
5. (i) $y = 2e^{4x} + e^{-3x}$ (ii) $y = (2 + 3x)e^{-3x}$
 (iii) $y = e^{2x} \sin 5x$ (iv) $y = e^{-\frac{x}{2}}(2 \cos 3x - \sin 3x)$
6. (i) $y = c_1e^x + c_2e^{2x} + 2x^2 + 6x + 7$ (ii) $y = e^{-x}(c_1 \sin x + c_2 \cos x) - \frac{7}{13} \sin 4x - \frac{4}{13} \cos 4x$
 (iii) $y = e^{-x}(c_1 \sin 3x + c_2 \cos 3x) + (\frac{1}{2}x + \frac{1}{10})e^{-2x}$ (iv) $y = c_1 \cos 2x + c_2 \sin 2x + 2x \sin 2x - x \cos 2x$
 (v) $y = c_1e^{2x} + c_2e^{-2x} + 2x^2e^{2x} - xe^{2x}$ (vi) $y = c_1 \sin x + c_2 \cos x - \frac{1}{4}x^2 \cos x + \frac{1}{4}x \sin x$
7. (i) $y = 3e^{3x} - 2e^{5x} + 3xe^{2x} + 4e^{2x}$ (ii) $y = (3x - 5)e^{-3x} + 3e^{-6x}$
 (iii) $y = 2e^{5x}(2 \sin 2x - \cos 2x + 1)$ (iv) $y = (3x^2 + x + 5)e^x + (2x - 4)e^{2x}$
8. (i) $y = c_1 \sin x + c_2 \cos x + x \sin x + \cos x \Upsilon |\cos x|$ (ii) $y = e^{-2x}(c_1 \sin x + c_2 \cos x) + xe^{-2x} \sin x + e^{-2x} \cos x \ln |\cos x|$
 (iii) $y = (c_1x + c_2)e^x - \frac{5}{36}x^3e^x + \frac{1}{6}x^3e^x \ln x$ (iv) $y = c_1 \sin x + c_2 \cos x + \frac{1}{2} \tan x + \frac{3}{2} \cos x \ln |\sec x + \tan x|$
 (v) $y = c_1e^{-x} + c_2e^{-2x} + (e^{-x} + e^{-2x}) \ln(1 + e^x)$ (vi) $y = c_1e^{-x} + c_2e^{-2x} + e^{-x} \ln|x| - e^{-2x} \int \frac{e^x}{x} dx$
9. (i) $y = c_1x + c_2xe^x - x^2$
 (ii) $y = c_1x + \frac{c_2}{x+1} + x^2 - \frac{2x^3+3x^2}{6(x+1)}$
 (iii) $y = c_1 \sin x + c_2x \sin x + \frac{1}{2}x^2 \sin x$

10. (i) $y = c_1 x^{\frac{1}{2}} + c_2 x^{\frac{3}{2}}$ (ii) $y = x^2 [c_1 \sin(\ln x^3) + c_2 \sin(\ln x^3)]$
 (iii) $y = (c_1 + c_2 \ln x) x^{\frac{1}{3}}$ (iv) $y = c_1 x^2 + c_2 x^4 - 2x^3$
 (v) $y = c_1 \sin(\ln x^2) + c_2 \cos(\ln x^2) + \frac{1}{5} x \ln x^2 - \frac{4}{25} x$ (vi) $y = x^2 [c_1 \sin(\ln x) + c_2 \cos(\ln x)] + 5x^2$
11. (i) $y = 2x^5 + 3x^{-2}$ (ii) $y = x^{-1} + x^2 - 2x + 4$
 (iii) $y = \frac{5}{3} x - 2x^2 + 3x^3 - \frac{23}{24} x^4$ (iv) $y = \frac{1}{18} x^3 + \frac{1}{12} x^{-2} - \frac{1}{6} \ln x + \frac{1}{36}$
12. $y = c_1(x+2)^3 + c_2(x+2)^{-1}$

Κεφάλαιο 4

1. $W = 6x^4 \neq 0$, $y = c_1 x + c_2 x^2 + c_3 x^4$
2. (i) $y = c_1 e^x + c_2 e^{-x} + c_3 e^{3x}$ (ii) $y = c_1 e^{-2x} + (c_2 x + c_3) e^{\frac{1}{2}x}$
 (iii) $y = c_1 e^x + c_2 \cos x + c_3 \sin x$ (iv) $y = (c_1 x^2 + c_2 x + c_3) e^{-\frac{1}{2}x}$
 (v) $y = c_1 e^x + c_2 e^{2x} + (c_3 x + c_4) e^{-x}$ (vi) $y = (c_1 x + c_2) e^{-2x} + e^{-x} (c_3 \sin \sqrt{2}x + c_4 \cos \sqrt{2}x)$
3. (i) $y = e^x - 2e^{2x} + e^{3x}$ (ii) $y = \frac{32}{9} e^{-x} - \frac{23}{9} e^{2x} + \frac{2}{3} x e^{2x}$
 (iii) $y = e^x + e^{2x} (2 \sin x - \cos x)$ (iv) $y = -2x + 2e^x$
4. $y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{4x} + e^{2x} [(c_5 + c_6 x + c_7 x^2) \sin 3x + (c_8 + c_9 x + c_{10} x^2) \cos 3x]$
5. $y = c_1 e^{-2x} + c_2 e^{-3x} + e^x (c_3 \sin 2x + c_4 \cos 3x)$
6. (i) $y = c_1 e^x + c_2 e^{-2x} + c_3 e^{-3x} + 3x^2 + x + 4$ (ii) $y = c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + 3x e^{2x} - e^{3x}$
 (iii) $y = c_1 + c_2 \sin x + c_3 \cos x + \frac{2}{3} x^3 - 4x - 2x \sin x$ (iv) $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} + \frac{1}{2} x^2 e^x + \frac{3}{4} x e^x + 4x e^{2x} + e^{4x}$
 (v) $y = (c_1 + c_2 x) e^x + c_3 e^{2x} + -\frac{1}{4} x^4 e^x - x^3 e^x + \frac{1}{2} x^2 e^x$ (vi) $y = c_1 + c_2 x + c_3 e^x + c_4 e^{-3x} - \frac{1}{2} x^4 - \frac{4}{3} x^3 - \frac{19}{8} x^2 + 2x^2 e^x - 9x e^x + \frac{1}{27} e^x$
7. (i) $y = \frac{7}{20} e^{-x} - \frac{31}{40} e^{2x} + \frac{3}{4} x e^x + \frac{5}{4} e^x - \frac{1}{10} \sin x$
 (ii) $y = (\frac{122}{9} - \frac{4}{3} x) e^x - \frac{4}{9} e^{4x} - 2x^2 - 9x - 15 + 3e^{2x}$
8. (i) $y = 2x$ (ii) $y = x^3 e^{-x}$
 (iii) $y = x(\cos 2x + \sin 2x)$ (iv) $y = \frac{1}{2} x e^x$
9. (i) $y = c_1 x + c_2 x^2 + c_3 x^3$ (ii) $y = c_1 x + c_2 x^2 + c_3 x^4 + c_4 x^{-1}$
 (iii) $y = c_1 x + c_2 x^2 + c_3 x^4 - \frac{1}{2} \Upsilon x - \frac{7}{8}$ (iv) $y = c_1 x^2 + c_2 x^4 + c_3 x^5 + \frac{1}{10} x^{-1}$

Κεφάλαιο 5

1. (i) $x_1 = 2 \cos t - 5 \sin t$, $x_2 = \cos t - 12 \sin t$
 (ii) $x_1 = 4e^{-t} - 3e^{-4t} + 2t + 1$, $x_2 = 2e^{-t} - 3e^{-4t} + t$
 (iii) $x_1 = (3t + 7)e^{-2t} - \sin t$, $x_2 = -(2t + 5)e^{-2t} + 2 \cos t$
2. (i) $x_1 = 2c_1 e^t + 2c_2 e^{-3t}$, $x_2 = 5c_1 e^t + c_2 e^{-3t}$
 (ii) $x_1 = -2c_1 e^{-\frac{7}{2}t} + 4c_2 e^{-t}$, $x_2 = c_1 e^{-\frac{7}{2}t} + 3c_2 e^{-t}$
 (iii) $x_1 = c_1 e^t + 2c_2 e^{2t} + c_3 e^{-t}$, $x_2 = 3c_2 e^{2t}$, $x_3 = c_2 e^{2t} + 2c_3 e^{-t}$
 (iv) $x_1 = 4c_1 e^{2t} - 7c_2 e^{5t} - 7c_3 e^{7t}$, $x_2 = 3c_2 e^{5t} + 5c_3 e^{7t}$, $x_3 = -5c_1 e^{2t} + 5c_2 e^{5t} + 5c_3 e^{7t}$
3. (i) $\mathbf{X} = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$
 (ii) $\mathbf{X} = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-5t}$
 (iii) $\mathbf{X} = c_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} e^{3t} + c_3 \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} e^{-2t}$
 (iv) $\mathbf{X} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} e^{-5t} + c_3 \begin{bmatrix} 2 \\ -2 \\ 11 \end{bmatrix} e^{6t}$
4. (i) $x_1 = -6e^{-2t} + e^{3t}$, $x_2 = -3e^{-2t} + e^{3t}$
 (ii) $x_1 = -2e^{-2t} + 3e^{\frac{1}{2}t}$, $x_2 = -e^{-2t} + 3e^{\frac{1}{2}t}$
 (iii) $x_1 = 2e^t - e^{2t}$, $x_2 = 2e^t$, $x_3 = -e^{2t}$
 (iv) $x_1 = -2 - e^t$, $x_2 = 2 + e^{-t}$, $x_3 = -e^{-t} + 2e^t$
5. (i) $x_1 = c_1(\sin t - \cos t) - c_2(\cos t + \sin t)$, $x_2 = 2c_1 \cos t + 2c_2 \sin t$
 (ii) $x_1 = c_1(\cos 3t + 3 \sin 3t) e^{5t} + c_2(\sin 3t - 3 \cos 3t) e^{5t}$, $x_2 = 2c_1 \cos 3t e^{5t} + 2c_2 \sin 3t e^{5t}$

- (iii) $x_1 = c_1 + c_2 \sin t - c_3 \cos t$, $x_2 = -c_2 \sin t + c_3 \cos t$, $x_3 = c_2 \cos t + c_3 \sin t$
 (iv) $x_1 = c_2(\sin 2t - 2 \cos 2t)e^t - c_3(2 \sin 2t + \cos 2t)e^t$, $x_2 = -2c_1e^{-3t} + 3c_2 \sin 2te^t - 3c_3 \cos 2te^t$, $x_3 = c_1e^{-3t} + 2c_2 \cos 2te^t + 2c_3 \sin 2te^t$

6. (i) $\mathbf{X} = c_1 \begin{bmatrix} 4 \cos 3t - 3 \sin 3t \\ 5 \cos 3t \end{bmatrix} + c_2 \begin{bmatrix} 4 \sin 3t + 3 \cos 3t \\ 5 \sin 3t \end{bmatrix}$
 (ii) $\mathbf{X} = c_1 \begin{bmatrix} 2 \cos 2t - 2 \sin 2t \\ \cos 2t \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \sin 2t + 2 \cos 2t \\ \sin 2t \end{bmatrix} e^{-t}$

(iii) $\mathbf{X} = c_1 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} \cos t \\ -\sin t \\ -\sin t \end{bmatrix} e^t + c_3 \begin{bmatrix} \sin t \\ \cos t \\ \cos t \end{bmatrix} e^t$

(iv) $\mathbf{X} = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{6t} + c_2 \begin{bmatrix} \sin 2t \\ 0 \\ 2 \cos 2t \end{bmatrix} e^{4t} + c_3 \begin{bmatrix} -\cos 2t \\ 0 \\ 2 \sin 2t \end{bmatrix} e^{4t}$

7. (i) $x_1 = (2 \sin t - \cos t)e^t$, $x_2 = 5e^t \sin t$

(ii) $x_1 = (2 \cos \frac{t}{2} - 3 \sin \frac{t}{2})e^t$, $x_2 = (-\cos \frac{t}{2} + 8 \sin \frac{t}{2})e^t$

(iii) $x_1 = 4 - \cos t - \sin t$, $x_2 = 2 - 2 \sin t$, $x_3 = -\sin t$

(iv) $x_1 = -e^t \sin t$, $x_2 = 2e^{-t} + e^t \cos t$, $x_3 = -e^t \cos t$

8. (i) $x_1 = c_1 + c_2(t + 1)$, $x_2 = 3c_1 + c_2(3t + 2)$

(ii) $x_1 = c_1e^{-t} + c_2te^{-t}$, $x_2 = c_1e^{-t} + c_2(t\frac{1}{5})e^{-t}$

(iii) $x_1 = c_1e^t + c_2e^{2t} + c_3e^{2t}$, $x_2 = c_1e^t + c_3e^{2t}$, $x_3 = c_1e^t + c_2e^{2t}$

(iv) $x_1 = 2c_1e^{8t} + c_3e^{-t}$, $x_2 = c_1e^{8t} - 2c_2e^{-t} - 2c_3e^{-t}$, $x_3 = 2c_1e^{8t} + c_2e^{-t}$

9. (i) $\mathbf{X} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} -\frac{1}{3} \\ 0 \end{bmatrix} \right) e^{2t}$

(ii) $\mathbf{X} = c_1 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{6t} + c_2 \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} t + \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} \right) e^{6t}$

(iii) $\mathbf{X} = c_1 \begin{bmatrix} -4 \\ -5 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{5t} + c_3 \left(\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} \frac{5}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} \right) e^{5t}$

(iv) $\mathbf{X} = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^t + c_2 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} e^{2t} + c_3 \left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \right) e^{2t}$

10. (i) $x_1 = -(t + 3)e^{\frac{t}{3}}$, $x_2 = (t + 4)e^{\frac{t}{3}}$

(ii) $x_1 = 1 + e^t$, $x_2 = 1 - e^t$, $x_3 = 1 + 2e^t$

(iii) $x_1 = 4 + (1 - t)e^t$, $x_2 = (t - 2)e^t$, $x_3 = 2 - e^t$

(iv) $x_1 = (1 - 4t - 2t^2)e^t$, $x_2 = (1 - 2t)e^t$, $x_3 = -(1 + t + t^2)e^t$

12. (i) $x_1 = 3c_1e^{8t} + c_2(3t + 2)e^{8t} + \frac{1}{2}$, $x_2 = c_1e^{8t} + c_2(t + 1)e^{8t} - \frac{1}{2}$

(ii) $x_1 = -c_1e^t \sin 4t - c_2e^t \cos 4t + \frac{4}{17} + e^{6t}$, $x_2 = c_1e^t \cos 4t - c_2e^t \sin 4t + t + \frac{1}{17} + e^{6t}$

(iii) $x_1 = 5c_1 \cos 2t + 5c_2 \sin 2t - 3 \cos t - \frac{1}{3} \sin t$, $x_2 = c_1(\cos 2t - 2 \sin 2t) + c_2(2 \cos 2t + \sin 2t) - \frac{2}{3} \cos t + \frac{1}{3} \sin t$

(iv) $x_1 = c_1e^t + c_2e^{2t} + c_3e^{5t} - \frac{3}{2}e^{4t}$, $x_2 = c_2e^{2t} + 2c_3e^{5t} - \frac{7}{2}e^{4t}$, $x_3 = 2c_3e^{5t} - 2e^{4t}$

13. (i) $x_1 = (-c_1 \sin t + c_2 \cos t)e^t + te^t \cos t$, $x_2 = (c_1 \cos t + c_2 \sin t)e^t + te^t \sin t$

(ii) $x_1 = c_1(\cos t - \sin t) + c_2(\cos t + \sin t) + 3 \sin t \cos t - \cos^2 t - 2 \sin^2 t + (\sin t - \cos t) \ln |\sec t + \tan t|$, $x_2 = c_1 \cos t + c_2 \sin t + \sin^2 t - \cos^2 t - \cos t \ln |\sec t + \tan t|$

(iii) $x_1 = c_1 + c_2e^{2t} - \frac{1}{4}e^{2t} + \frac{1}{2}te^{2t}$, $x_2 = -c_1 + c_2e^{2t} - e^t + \frac{1}{4}e^{2t} + \frac{1}{2}te^{2t}$, $x_3 = c_3e^{3t} + \frac{1}{2}t^2e^{3t}$

(iv) $x_1 = c_1e^t + c_2e^{2t} + c_3e^{2t} - \frac{1}{2}t - \frac{3}{4} + 2e^t + 2te^t$, $x_2 = c_1e^t + c_2e^{2t} - t - 1 + 2e^t + 2te^t$, $x_3 = c_1e^t + c_3e^{2t} - \frac{1}{2}t - \frac{3}{4} + 2te^t$

14. (i) $x_1 = -c_1e^{-t} - 3c_2e^t - 1$, $x_2 = c_1e^{-t} + c_2e^t + 3$

(ii) $x_1 = c_1e^{-2t} + c_2e^{4t} - \frac{1}{4}t^2 + \frac{1}{4}t - 2$, $x_2 = -c_1e^{-2t} + c_2e^{4t} + \frac{3}{4}t^2 - \frac{1}{4}t + \frac{3}{4}$

(iii) $x_1 = c_1e^{3t} + c_2e^{7t} + \frac{5}{36}e^t$, $x_2 = -3c_1e^{3t} + 9c_2e^{7t} - \frac{19}{4}e^t$

(iv) $x_1 = c_1e^{5t} + c_2e^{5t} + c_3e^{-5t} - 8$, $x_2 = c_2e^{5t} + 2$, $x_3 = c_1e^{5t} + c_2e^{5t} - c_3e^{-5t} - 1$

15. (i) $x_1 = 2e^{-t} - 2e^{4t} - 1 + e^{2t}$, $x_2 = 2e^{-t} - e^{4t} + e^{2t}$

(ii) $x_1 = -3e^{-2t} + (t + 1)e^{3t}$, $x_2 = -3e^{-2t} - 2(t + 1)e^{3t}$

(iii) $x_1 = -2e^{-2t} + e^{-t} + 4e^t$, $x_2 = 2e^{-2t}$, $x_3 = -2e^{-2t} - 1 + 2e^t$

(iv) $x_1 = -2e^{-t} + e^{2t}$, $x_2 = 2e^t + 2e^{2t}$, $x_3 = e^{-t} - 2e^t + 2e^{2t}$

16. (i) $e^{At} = \begin{bmatrix} t+1 & t & t \\ t & t+1 & t \\ -2t & -2t & -2t+1 \end{bmatrix}$, $x_1 = c_1(t+1) + c_2t + c_3t$, $x_2 = c_1t + c_2(t+1) + c_3t$, $x_3 = -2c_1t - 2c_2t + c_3(1-2t)$

(ii) $e^{At} = \begin{bmatrix} 1 & 0 & 0 \\ 3t & 1 & 0 \\ \frac{3}{2}t^2 + 5t & t & 1 \end{bmatrix}$, $x_1 = c_1$, $x_2 = 3c_1t + c_2$, $x_3 = c_1(\frac{3}{2}t^2 + 5t) + c_2t + c_3$

Κεφάλαιο 6

1. (i) $\frac{1}{s^2}(1 - e^{-s})$ (ii) $\frac{1 + e^{-s\pi}}{1 + s^2}$ (iii) $\frac{1}{s^2 + 2s + 1}$
- (iv) $\frac{s^2 - 1}{(s^2 + 1)^2}$ (v) $\frac{8}{s^3} - \frac{15}{s^2 + 9}$ (vi) $\frac{1}{2(s-2)} - \frac{1}{2s}$
- (vii) $\frac{2}{s^2 + 16}$ (viii) $\frac{1}{2} \left(\frac{3}{s^2 + 9} - \frac{1}{s^2 + 1} \right)$
2. (i) $\frac{6}{(s+2)^4}$ (ii) $\frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^4}$
- (iii) $\frac{e^{-2s}}{s^2} + 2\frac{e^{-2s}}{s}$ (iv) $\frac{s}{s^2 + 4}e^{-\pi s}$
4. (i) $f(t) = 1 - \mathcal{U}(t-4) + \mathcal{U}(t-5)$, $\mathcal{L}\{f(t)\} = \frac{1}{s}(1 - e^{-4s} + e^{-5s})$
- (ii) $f(t) = t^2\mathcal{U}(t-1)$, $\mathcal{L}\{f(t)\} = 2\frac{e^{-s}}{s^3} + 2\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}$
- (iii) $f(t) = t - t\mathcal{U}(t-2)$, $\mathcal{L}\{f(t)\} = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - 2\frac{e^{-2s}}{s}$
- (iv) $f(t) = \sin t - \sin t\mathcal{U}(t-2\pi)$, $\mathcal{L}\{f(t)\} = \frac{1 - e^{-2\pi s}}{s^2 + 1}$
5. $\frac{1 - (s+1)e^{-s}}{s^2(1 - e^{-2s})}$
6. (i) $\frac{6}{s^5}$ (ii) $\frac{48}{s^8}$ (iii) $\frac{s-1}{(s+1)[(s-1)^2 + 1]}$
7. (i) $\frac{s^2 - a^2}{(s^2 + a^2)^2}$ (ii) $\frac{6s^2 - 2}{(s^2 + 1)^3}$ (iii) $\frac{6s^4 - 36s^2 + 6}{(s^2 + 1)^4}$ (iv) $\ln\left(\frac{s+b}{s+a}\right)$ (v) $\frac{1}{2}\ln\left(\frac{s+1}{s-1}\right)$
9. (i) $1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$ (ii) $\frac{1}{4}e^{-t/4}$ (iii) $\frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$
- (iv) $2\cos 3t - 2\sin 3t$ (v) $\frac{1}{4}\sinh 4t$ (vi) $\frac{1}{3}\sin t - \frac{1}{6}\sin 2t$
10. (i) $\frac{1}{2}t^2e^{-2t}$ (ii) $e^{-t}(1-t)$
- (iii) $5-t-e^{-t}(5+4t+\frac{3}{2}t^2)$ (iv) $\frac{1}{2}(t-2)^2\mathcal{U}(t-2)$
- (v) $\mathcal{U}(t-1)(1 - e^{-(t-1)})$ (vi) $\frac{1}{2}t\sin t$
11. (i) $1 - e^{-t}$ (ii) $-\frac{1}{3}e^{-t} + \frac{1}{3}e^{2t}$
- (iii) $\frac{1}{4}t\sin 2t$ (iv) $\frac{1}{4}\sin te^{-2t} - \frac{1}{2}te^{-2t}\cos t + \frac{1}{4}\sin t$
12. (i) $\frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t}$ (ii) $\frac{1}{5}e^{-t}(4\cos t - 3\sin t) - \frac{4}{5}e^{-3t}$ (iii) $\frac{1}{2}\sin t \sinh t$
13. (i) $y(t) = \frac{1}{9}t + \frac{2}{27} - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$
- (ii) $y(t) = \frac{1}{2}(e^t \sin t - e^t \cos t + 1)$
- (iii) $y(t) = -t + \frac{1}{2}\sin t + \frac{1}{4}e^t - \frac{1}{4}e^{-t}$
- (iv) $y(t) = \cos 2t - \frac{1}{6}\sin 2(t-2\pi)\mathcal{U}(t-2\pi) + \frac{1}{3}\sin(t-2\pi)\mathcal{U}(t-2\pi)$
- (v) $y(t) = e^{-2t}\cos 3t + \frac{2}{3}e^{-2t}\sin 3t + \frac{1}{3}e^{-2(t-\pi)}\sin 3(t-\pi)\mathcal{U}(t-\pi) + \frac{1}{3}e^{-2(t-3\pi)}\sin 3(t-3\pi)\mathcal{U}(t-3\pi)$
14. (i) $y(t) = (e-1)e^{-t} + (e+1)te^{-t}$
- (ii) $y(t) = \frac{1}{3}t^3 + ct^2$
15. (i) $x(t) = -\frac{1}{2}t - \frac{3}{4}\sqrt{2}\sin(\sqrt{2}t)$, $y(t) = -\frac{1}{2}t + \frac{3}{4}\sqrt{2}\sin(\sqrt{2}t)$
- (ii) $x(t) = 1 + t + \frac{1}{2}t^2 - e^{-t}$, $y(t) = -\frac{1}{3}(1 - e^{-t} - te^{-t})$
16. (i) $y = 2e^{2t} - 3e^{-t} - e^{-t}\sin 3t + e^{-t}\cos 3t$
- (ii) $y = e^{-2t} - e^{-3t} - 2te^{-3t}$

(iii) $y2e^{-t} - 3e^{-2t} + \cos t + 3 \sin t$

(iv) $y = (3 - 4t)e^t + \sin t - 3 \cos t$

17. (i) $y = \begin{cases} -t + 2\pi + \frac{1}{2} \sin 2t + (2 - 2\pi) \cos 2t, & 0 < t < 2\pi \\ (2 - 2\pi) \cos 2t, & t > 2\pi \end{cases}$

(ii) $y = \begin{cases} e^{3t} - \cos t - 3 \sin t, & 0 < t < 2\pi \\ e^{3t}(1 - e^{-6\pi}), & t > 2\pi \end{cases}$

18. (i) $y(t) = t^2 - t$ (ii) $y(t) = 3 - t$

19. $x(t) = (t + 4)e^{-t} + 1$, $y(t) = e^{-t} + 1$, $z(t) = (t + 2)e^{-t}$

20. $y(t) = \int_0^t e^{-2u} \sin uf(t - u) du$
 $y(t) = \frac{1}{10}e^t - \frac{1}{10}e^{-2t}(3 \sin t + \cos t)$

Κεφάλαιο 7

1. (i) $y_1(x) = c_0 \sum_{n=1}^{\infty} \frac{1}{n} x^n$, $y_2(x) = 0$

(ii) $y_1(x) = c_0 \sum_{n=1}^{\infty} x^{2n}$, $y_2(x) = c_1 \sum_{n=0}^{\infty} x^{2n+1}$

(iii) $y_1(x) = c_0 \left(1 + \frac{1}{4}x^2 - \frac{7}{4.4!}x^4 + \frac{23.7}{8.6!}x^6 - \dots \right)$
 $y_2(x) = c_1 \left(x - \frac{1}{6}x^3 + \frac{14}{2.5!}x^5 - \frac{34.14}{4.7!}x^7 + \dots \right)$

(iv) $y_1(x) = c_0 \left(1 + \frac{1}{2}x + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \dots \right)$, $y_2(x) = c_1 \left(x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \dots \right)$

2. (i) $y(x) = 8x - 2e^x$

(ii) $y(x) = 3 - 12x^2 + 4x^4$

3. (i) $x = -2$, $t = x + 2$: $ty'' + (t - 2)y' - 3y = 0$

(ii)

(iii) $x_1 = 3$, $t = x - 3$: $(t^2 + 6t)y'' + (2t + 7)y' + (t + 3)y = 0$

$x_2 = -3$, $t = x + 3$: $(t^2 - 6t)y'' + (2t - 5)y' + (t - 3)y = 0$

(iv)

4. (i) $x = 0$, ασύνηθες μη ομαλό σημείο(ii) $x = -3$, σύνηθες μη ομαλό σημείο $x = 3$ ασύνηθες μη ομαλό σημείο(iii) $x = 0$, $2i$, $-2i$, σύνηθες μη ομαλά σημεία(iv) $x = -3, 2$, σύνηθες μη ομαλά σημεία(v) $x = 0$, ασύνηθες μη ομαλό σημείο $x = -5, 5, 2$, σύνηθες μη ομαλά σημεία

5. (i) $r_1 = 1$, $r_2 = -\frac{1}{2}$, $y_1(x) = x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4 + \frac{1}{24}x^5 - \dots$

$y_2(x) = x^{-\frac{1}{2}}(1 + 2x - 4x^2 + \frac{8}{3}x^3 - \frac{16}{15}x^4 + \dots)$, $-\infty < x < \infty$

(ii) $r_1 = \frac{1}{2}$, $r_2 = 0$, $y_1(x) = x^{\frac{1}{2}}(1 + \frac{2}{3}x + \frac{1}{30}x^2 - \frac{1}{35}x^3 - \frac{19}{7560}x^4 + \dots)$

$y_2(x) = 1 + 2x + \frac{1}{2}x^2 - \frac{1}{15}x^3 - \frac{19}{840}x^4 + \dots$, $-\infty < x < \infty$

(iii) $r_1 = \frac{1}{3}$, $r_2 = -1$, $y_1(x) = x^{\frac{1}{3}}(1 + \frac{2}{21}x - \frac{1}{630}x^2 + \frac{2}{36855}x^3 - \frac{1}{505440}x^4 + \dots)$

$y_2(x) = x^{-1} - 2 - \frac{1}{2}x$, $-\infty < x < \infty$

(iv) $r_1 = 0$, $r_2 = -\frac{2}{3}$, $y_1(x) = 1 - x + \frac{1}{2}x^2 - \frac{7}{54}x^3 + \frac{29}{2376}x^4 + \dots$

$y_2(x) = x^{-\frac{3}{2}}(1 - \frac{13}{4}x + \frac{127}{32}x^2 - \frac{3013}{1152}x^3 + \frac{87167}{92160}x^4 - \dots)$, $-2 < x < 2$

(v) $r_1 = \frac{1}{2}$, $r_2 = 0$, $y_1(x) = x^{\frac{1}{2}}(1 + \frac{1}{5}x - \frac{1}{60}x^2 - \frac{13}{180}x^3 - \frac{131}{12960}x^4 + \dots)$

$y_2(x) = 1 + x + \frac{1}{6}x^2 - \frac{11}{90}x^3 - \frac{131}{2520}x^4 + \dots$, $-\infty < x < \infty$

6. (i) $r_1 = r_2 = 0$, $y_1(x) = 1 - \frac{1}{2}x^2 + \frac{1}{9}x^3 + \frac{1}{24}x^4 - \frac{7}{450}x^5 + \dots$

$y_2(x) = y_1(x) \ln x + (-x + \frac{3}{4}x^2 + \frac{1}{27}x^3 - \frac{37}{288}x^4 + \frac{299}{13500}x^5 + \dots)$, $x > 0$

(ii) $r_1 = r_2 = 1$, $y_1(x) = x(1 + x - \frac{1}{9}x^3 - \frac{1}{144}x^4 + \frac{1}{240}x^5 + \dots)$

$y_2(x) = y_1(x) \ln x + x(-2x - \frac{1}{2}x^2 + \frac{13}{54}x^3 + \frac{43}{864}x^4 - \frac{67}{7200}x^5 + \dots)$, $x > 0$

(iii) $r_1 = r_2 = 0$, $y_1(x) = 1 + x + \frac{1}{4}x^2 + \frac{5}{36}x^3 + \frac{23}{576}x^4 + \dots$

$y_2(x) = y_1(x) \ln x + (-2x - \frac{1}{2}x^2 - \frac{7}{27}x^3 - \frac{287}{3456}x^4 - \frac{15631}{432000}x^5 + \dots)$, $x > 0$

(iv) $r_1 = r_2 = -1$, $y_1(x) = x^{-1}(1 + x - \frac{1}{2}x^2 + \frac{5}{18}x^3 - \frac{5}{36}x^4 + \dots)$

- $y_2(x) = y_1(x) \ln x + x^{-1}(-5x + \frac{9}{4}x^2 - \frac{137}{108}x^3 + \frac{563}{864}x^4 - \frac{6361}{21600}x^5 + \dots, x > 0$
 (v) $r_1 = r_2 = 0, y_1(x) = 1 - 2x + \frac{3}{4}x^2 + \frac{3}{64}x^4 + \frac{160}{3}x^5 + \dots$
 $y_2(x) = y_1(x) \ln x + (3x - \frac{5}{2}x^2 + \frac{1}{36}x^3 - \frac{145}{1152}x^4 - \frac{17}{400}x^5 + \dots), x > 0$
- 7.** (i) $r_1 = \frac{1}{2}, r_2 = -\frac{1}{2}, y_1(x) = x^{\frac{1}{2}}(1 - \frac{1}{4}t + \frac{1}{24}x^2 - \frac{1}{192}x^3 + \frac{1}{1920}x^4 - \dots)$
 $y_2(x) = x^{-\frac{1}{2}}, -\infty < x < \infty$
 (ii) $r_1 = 0, r_2 = -2, y_1(x) = 1 + \frac{4}{3}x + \frac{2}{3}x^2 + \frac{1}{15}x^3 + \frac{1}{30}x^4$
 $y_2(x) = x^{-2}(1 + 8x + \frac{1}{3}x^3 + \frac{7}{6}x^4 + \frac{1}{72}x^6 + \dots), -\infty < x < \infty$
 (iii) $r_1 = 1, r_2 = -3, y_1(x) = x(1 - \frac{2}{5}x + \frac{2}{15}x^2 - \frac{4}{105}x^3 + \frac{1}{105}x^4 + \dots)$
 $y_2(x) = x^{-3}(1 - 2x + 2x^2 - \frac{4}{3}x^3), -\infty < x < \infty$
 (iv) $r_1 = 3, r_2 = 1, y_1(x) = x^3(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 + \dots)$
 $y_2(x) = -4y_1(x) \ln x + x(1 + 2x - 8x^3 + 12x^4 - \frac{88}{9}x^5 + \dots), 0 < x < \infty$
 (v) $r_1 = 2, r_2 = -1, y_1(x) = x^2(1 - \frac{5}{4}x + \frac{3}{4}x^2 - \frac{7}{24}x^3 + \frac{1}{12}x^4 + \dots)$
 $y_2(x) = -2y_1(x) \ln x + x^{-1}(1 + x + \frac{3}{2}x^2 - \frac{21}{8}x^4 + \frac{19}{8}x^5 + \dots), 0 < x < \infty$
- 8.** (i) $y(x) = c_1 x^{\frac{3}{2}} \left(1 - \frac{2}{5}x + \frac{2^2}{7.5.2}x^2 - \frac{2^3}{9.7.5.3!}x^3 + \dots \right) + c_2 \left(1 + 2x - 2x^2 + \frac{2^3}{3.3!}x^3 - \dots \right)$
 (ii) $y(x) = c_1 x^{\frac{1}{3}} \left(1 + \frac{1}{3}x + \frac{1}{3^2.2}x^2 + \frac{1}{3^3.3!}x^3 + \dots \right) + c_2 \left(1 + \frac{1}{2}x + \frac{1}{5.2}x^2 + \frac{1}{8.5.2}x^3 + \dots \right)$
 (iii) $y(x) = c_1 x \left(1 + \frac{1}{5}x + \frac{1}{5.7}x^2 + \frac{1}{5.7.9}x^3 + \dots \right) + c_2 x^{-\frac{1}{2}} \left(1 + \frac{1}{2}x + \frac{1}{2.4}x^2 + \frac{1}{2.4.6}x^3 + \dots \right)$
 (iv) $y(x) = \frac{1}{x} (c_1 \cosh x + c_2 \sinh x)$
 (v) $y(x) = c_1 e^x + c_2 e^x \left(\ln x - x + \frac{1}{2.2!}x^2 - \frac{1}{3.3!}x^3 + \frac{1}{4.4!}x^4 - \dots \right)$
 (vi) $y(x) = c_1 x^2 + c_2 \left(\frac{1}{2}x^2 \ln x - \frac{1}{2} + x - \frac{1}{3!}x^3 + \dots \right)$

Επιπρόσθετες Ασκήσεις

- 2** (i) $x^2 y'' - 4xy' + 6y = 0$ (ii) $xy' + 2y = 0$
- 3** (i) $y = cx$ (ii) $y = ce^{-\sqrt{1+x^2}} - 1$ (iii) $y = 1 - ce^{-x - \frac{1}{3}x^3}$ (iv) $y = \ln(\sec x + c)$
- 4** (i) $\ln|x| - \frac{1}{3} \left(\frac{y}{x} \right)^3 = c$ (ii) $y + \sqrt{x^2 + y^2} = c$ (iii) $y^2 - 4xy + x^2 = c$ (iv) $y^3 = x^3(c - 6 \ln x)$
- 5** (i) $y = e^{-2x} + ce^{-3x}$ (ii) $y = e^{-x} \sin(e^x) + ce^{-x}$ (iii) $y = \frac{1}{2} \sinh x + \frac{1}{2} x \operatorname{sech} x + c \operatorname{sech} x$ (iv) $y = cx + x \sin x$
- 6** (i) $y = -\ln(3 - \frac{1}{2}x^2)$ (ii) $y = x \cos x - \sin x \cos^2 x + \cos x$
- 7** (i) $y = Ae^{-x} + Be^{-2x}$ (ii) $y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2}x + A \sin \frac{\sqrt{3}}{2}x \right)$ (iii) $y = (A + Bx)e^{3x}$ (iv) $y = e^{2x}(A \cos 3x + B \sin 3x)$
- 8** $y = (A + Bx)e^x + Ce^{-x}$
- 9** (i) $y = 2e^x - e^{-3x}$ (ii) $y = 4e^{-x} + e^{7x}$ (iii) $y = -e^{-2x}(3 \cos x + 6 \sin x)$ (iv) $e^{3x}(2 \sin 2x - \cos 2x)$
- 10** (i) $y = c_1 e^{-2x} + c_2 e^{-x} - \frac{3}{8} \cos 2x - \frac{1}{4} \sin 2x$ (ii) $y = c_1 \cos x + c_2 \sin x - \frac{1}{2}x \cos x$ (iii) $y = c_1 e^x + c_2 e^{2x} + x e^{2x}$
 (iv) $e^{-2x}(c_1 \cos \sqrt{5}x + c_2 \sin \sqrt{5}x) + \frac{1}{9}x^2 + \frac{19}{81}x - \frac{94}{729}$ (v) $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{8}x \cos 2x$ (vi) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}x \sin 2x$
- 11** (i) $y = c_1 e^x + c_2 e^{-x} - 1 + \frac{1}{2}x e^x$ (ii) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4} + \frac{1}{4}x + \frac{1}{3} \sin x$ (iii) $y = (c_1 + c_2 x)e^x + \frac{1}{4}x^2 e^x + \frac{1}{8}e^{-x}$ (iv) $y = c_1 \cos x + c_2 \sin x + 6 - 2 \cos 2x$
- 12** (i) $y = c_1 \cos x + c_2 \sin x - \cos x \ln|\sec x + \tan x|$ (ii) $y = (c_1 + c_2 x)e^x + x e^x \ln|x|$ (iii) $y = c_1 \cos x + c_2 \sin x + 3 \cos^2 x - \cos^4 x + \sin^4 x$ (iv) $y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \ln|\sin x|$ (v) $y = c_1 \cos x + c_2 \sin x + x \cos x - \sin x \ln|\cos x|$ (vi) $y = (c_1 + c_2 x)e^{-x} - e^{-x} \ln|x|$ (vii) $y = (c_1 + c_2 x)e^{-2x} + (x - 2)e^{-x}$
 (viii) $y = c_1 \cos x + c_2 \sin x - 1 + \sin x \ln|\sec x + \tan x|$
- 13** $y = c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + 3x + \frac{3}{2} + \frac{1}{5} \sin x + \frac{1}{10} \cos x$
- 15** (i) $y = \left(\frac{c}{x^2} - \frac{2}{3}x \right)^{-\frac{1}{2}}$ (ii) $y = \left(ce^{\frac{x^2}{2}} - 1 \right)^2$
- 16** $a = \pm 1, b = 1, y = e^x + \frac{2}{2ce^x - e^{-x}}$
- 17** (i) $y = \frac{1}{x}(c_1 \cos(\ln x) + c_2 \sin(\ln x))$ (ii) $y = c_1 x^{1+\sqrt{3}} + c_2 x^{1-\sqrt{3}}$ (iii) $y = c_1 x^2 + \frac{c_2}{x^3}$ (iv) $y = x(c_1 \cos(\sqrt{3} \ln x) + c_2 \sin(\sqrt{3} \ln x)) + \frac{3}{13} \cos(\ln x) - \frac{2}{13} \sin(\ln x)$

18 (i) $y = e^{2x} + e^{-2x} + c_1x + c_2$ (ii) $y = c_1e^x + c_2e^{2x} + c_3$ (iii) $y = \frac{1}{2}(\ln c_1x)^2 + c_2$ (iv) $y = (c_1 + c_2x)e^x + c_3e^{-2x} + \frac{1}{6}x^2e^x$

19 $y = (c_1x + c_2)^2$

20 $y = \tan^{-1}\left(\frac{x^2}{3} + \frac{c}{x}\right)$

23. $\left(\frac{x^2}{x+1}\right)(x^2 - 3x + c)$

24. $y = \left(\frac{1}{4}x + \frac{5}{16}\right)e^{-2x} - \frac{1}{3}e^{-4x} + \frac{1}{48}e^{2x}(\cdot)$

25. $y = \frac{1}{x^2}e^{-x}(A \cos x + B \sin x) + \frac{1}{2}xe^{-x} \sin x$

28. (i) $\ln(e^x + c) = -\frac{1}{\cos y} + c$, $\cos y = 0$ (ii) $y^2(\ln y - \frac{1}{2}) = e^{x^2}(x^2 - 1) + c$ (iii) $e^{-x}(x + 1) = \frac{1}{2\cos^2 y} + c$, $\cos y = 0$ (iv) $(x - 1)e^x + \ln(y^2 + 1) + \tan^{-1} y = c$ (v) $e^{-x^2} + \frac{1}{y^2} = c$, $y = 0$

29. (i) $y = -\cos x + c \sin x$ (ii) $y = \frac{1}{\cos^3 x}(\sin x - \frac{1}{3}\sin^3 x + c)$ (iii) $y = (1 + \cos x)(x - \sin x + c)$ (iv) $(1 + x)(\frac{1}{2}x^2 + c)$ (v) $y = x^2 - 2x + 2 + ce^{-x}$

30. (i) $y^{-3} = -\frac{3}{4}x(2 \ln x - 1) + \frac{c}{x}$, $y = 0$ (ii) $y^{-2} = \frac{1}{x}((6 + ce^{-x}))$, $y = 0$ (iii) $x^2y^2 = -2x \cos x + 2 \sin x + c$ (iv) $y^4 = 8x^3 + cx^4$ (v) $(y - 2x^{\frac{3}{2}})^2 = cyx^6$, $y = 0$

31. (i) $x = -(12t + 13)e^t + e^{-3t} + 16e^{2t}$, $y = -2(4t + 3)e^t - 2e^{-3t} + 8e^{2t}$

(ii) $x = 5e^t(1 - \cos t + \sin t)$, $y = 5e^t(1 - \cos t)$

(iii) $x = 22e^{-2t} - 33e^{-t} - 3(2t - 3)e^t + 3e^{3t}$, $y = 11e^{-2t} - 12(t + 1)e^t + 4e^{3t}$

(iv) $x = 3t + 2 + 2e^t - 3e^{2t} + 8e^{3t}$, $y = -6t - 1 - 4e^t + 8e^{3t}$, $z = 3t - 1 + e^t - e^{2t} + 2e^{3t}$

(v) $x = 12 \cos 2t - 24 \sin 2t - 10e^{-2t} + 30e^{2t} - 32e^t$, $y = -12 \cos 2t - 6 \sin 2t + 15e^{-2t} + 45e^{2t} - 48e^t$