ΑΠΑΝΤΗΣΕΙΣ ΑΣΚΗΣΕΩΝ

Κεφάλαιο 1

2 (i) $a = -4$ (ii) $a \in \mathbb{R} - \{-4, 4\}$ (iii) $a = 4$

3

(i) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 7/8 & 3/4 & 0 \\ 0 & 1 & 5/8 & -11/4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4 (i) αδύνατο (ii) $x_2 = t,\ x_1 = 3 + 2t$ (iii) $x_1 = -4,\ x_2 = 2,\ x_3 = 7$
(iv) $x_1 = \frac{8}{5} - \frac{3}{5}s - \frac{3}{5}t,\ x_2 = \frac{1}{10}(1 + 4s - t),\ x_3 = s,\ x_4 = t$
6 (i) $x_1 = 3,\ x_2 = -1,\ x_3 = 2$ (ii) $x_1 = 2,\ x_2 = -\frac{4}{11},\ x_3 = \frac{21}{11}$
(iii) $x_1 = x_2 = x_3 = x_4 = 1$ (iv) $x_1 = 1,\ x_2 = 0,\ x_3 = -1, x_4 = -2$
7 (i) $x_1 = x_2 = x_3 = 0$ (ii) $x_1 = 7s - 5t,\ x_2 = -6s + 4t,\ x_3 = 2s,\ x_4 = 2t$
(iii) $x_1 = x_2 = x_3 = x_4 = 0$
8 (i) $k = 2$ ή $k = 4$ (ii) $k = 9$

Κεφάλαιο 2

1 (i) $\begin{bmatrix} 8 & 2 & 1 \\ 9 & 5 & 9 \\ 4 & -3 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 3 & -8 \\ 10 & -3 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ (iii) $\begin{bmatrix} 18 & 1 & 10 \\ 8 & 13 & 16 \\ 7 & -6 & 15 \end{bmatrix}$

$D = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

2 (i) $[3]$ (ii) $\begin{bmatrix} 3 & -3 & 6 \\ 2 & -2 & 4 \\ 1 & -1 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ (iv) Δεν ορίζεται (v) Δεν ορίζεται

(vi) $\begin{bmatrix} 9 \\ 0 \end{bmatrix}$ (vii) $\begin{bmatrix} 11 \\ 8 \\ 1 \end{bmatrix}$ (viii) $\begin{bmatrix} 8 & -4 & 19 \end{bmatrix}$ (ix) Δεν ορίζεται (x) $\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

(xi) $\begin{bmatrix} 1 & 6 & 2 \\ 5 & 2 & 11 \\ 6 & 0 & 11 \end{bmatrix}$ (xii) $\begin{bmatrix} 21 & 14 & 1 \\ 14 & 13 & 5 \\ 1 & 5 & 6 \end{bmatrix}$ (xiii) $[50]$ (xiv) $[6]$ (xv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3 $\mathbf{AB} = \begin{bmatrix} 4 & 5 & 5 \\ 3 & 3 & 3 \\ 7 & 8 & 6 \end{bmatrix}$ $\mathbf{BA} = \begin{bmatrix} 5 & 6 & 2 \\ 7 & 6 & 4 \\ 5 & 5 & 2 \end{bmatrix}$

6 (i) μη-αντιστρέψιμος (ii) $\frac{1}{11}\begin{bmatrix} 2 & 1 & 1 \\ 7 & -2 & -2 \\ 8 & 4 & -7 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ -1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$
(iv) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{bmatrix}$ (v) μη-αντιστρέψιμος (vi) $\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{3}{2} & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{4}{5} & \frac{2}{5} & -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}$

8 (i) $x_1 = 1,\ x_2 = 2,\ x_3 = -3$ (ii) $x_1 = -1,\ x_2 = 1,\ x_3 = -2$
(iii) $x_1 = -2,\ x_2 = 1,\ x_3 = 4$

9 $x = \frac{5}{9},\ y = 9,\ z = \frac{1}{3}$

10 $\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

11 (i) $\begin{bmatrix} 2 \\ -\frac{3}{2} \\ \frac{5}{2} \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & -3 & -3 \\ 3 & 1 & -4 \\ -3 & -3 & 9 \end{bmatrix}$

Κεφάλαιο 3

1 (i) 11 (ii) -12 (iii) -6 (iv) 4

2 $x = \frac{1}{4}(3 \pm \sqrt{33})$

4 (i) -17 (ii) 33 (iii) -21 (iv) 6 (v) (vi)

6 (i) -6 (ii) 72 (iii) -6 (iv) 18

7 (i) -189 (ii) $-\frac{8}{7}$ (iii) $-\frac{1}{56}$ (iv) 49

9 $\det(\mathbf{A}) = 152$

10 (i) -10 (ii) -32 (iii) -42 (iv) 50 (v) 263

11 (i) $\begin{bmatrix} -1/14 & 2/7 \\ 2/7 & -1/7 \end{bmatrix}$ (ii) $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ (iii) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$
(iv) $\begin{bmatrix} 3/8 & -1/4 & 1/4 \\ 1/8 & 1/4 & -1/4 \\ -3/8 & 1/4 & 3/4 \end{bmatrix}$ (v) $\begin{bmatrix} 3/8 & -1/4 & 1/8 \\ 3/16 & 3/8 & 1/16 \\ -1/16 & -1/8 & 5/16 \end{bmatrix}$ (vi) μη-αντιστρέψιμος

13 (i) 6 (ii) 0 (iii) -6 (iv) -12

14 (i) $x = 3, y = -1, z = 2$ (ii) $x = 5, y = 1, z = 1$ (iii) $x = 1, y = 1, z = -3$
(iv) $x_1 = x_2 = x_3 = x_4 = 1$ (v) $x_1 = 1, x_2 = 0, x_3 = -1, x_4 = -2$

15 $x_1 = x_2 = x_3 = x_4 = 0$

Κεφάλαιο 4

1 (i) $(-1, 1, 4)$ (ii) $(5, -5, -2, 3)$ (iii) $(0, 0)$ (iv) δεν ορίζεται (v) $(-1, 0, 7)$
(vi) $(-3, 6, -3, 0)$ (vii) $(2, -1, 5)$

2 $x = \frac{5}{2},\ y = 5$

3 $x = 4,\ y = -7,\ z = 5$

6 (i) -8 (ii) -35 (iii) 8

7 (i) $k = -2$ (ii) $k = -1$

8 (i) 13 (ii) $\sqrt{83}$

9 $k = 2$ ή -4

10 (i) $\sqrt{53}$ (ii) 13

11 $k = \pm 3$

12 (i) Όχι (ii) Ναι (iii) Όχι (iv) Ναι

13 (i) $(-\frac{3}{5}, \frac{4}{5})$ (ii) $(\frac{1}{3}, -\frac{2}{3}, 0, \frac{2}{3})$

15 $\theta = \cos^{-1}(\frac{1}{21})$

16 (i) $4/\sqrt{29}$

(ii) $\sqrt{564/29}$

17 $c_1 = \frac{3}{7},\ c_2 = -\frac{1}{3},\ c_3 = \frac{1}{21}$

18 $E = \frac{\sqrt{26}}{2},\ h = \frac{\sqrt{26}}{3}$

21 (i) Το xy-επίπεδο στο (-2,10,0), το xz-επίπεδο στο (-2,0,-5) και δεν τέμνει το yz-επίπεδο.

(ii) (5/4,9/4,1/2)

(iii) (0,4,-2) και (4,0,6)

23 (i) τέμνονται στο (1,-1,2)

(ii) τέμνονται στο (-17,-1,1)

(iii) δεν τέμνονται, δεν είναι παράλληλες

(iv) δεν τέμνονται, δεν είναι παράλληλες

24 (i) $2\sqrt{5}$

(ii) $4\sqrt{\frac{10}{11}}$

(iii) $\sqrt{\frac{35}{6}}$

25 (i) (2,0,3)

(ii) $\theta \approx 48^o$

(iii) $x = 2 + 3t,\ y = 7t,\ z = 3 + t$

26 (i) $x + 5y + 3z = -6$

(ii) $x + y - 3z = 6$

(iii) $7x - y - 3z = 5$

27 $x - 4y + 4z = -9$

28 (i) $D = \frac{9}{7}$

(ii) $D = \frac{11}{\sqrt{116}}$

(iii) $D = \sqrt{6}$

Κεφάλαιο 5

4 $-6\mathbf{e}_1 + 3\mathbf{e}_2 + 2\mathbf{e}_3$

5 αδύνατο

6 $k = -8$

7 (i) Ναι (ii) Όχι

9 (i) γρ. ανεξάρτητα (ii) γρ. ανεξάρτητα (iii) γρ. εξαρτημένα (iv) γρ. εξαρτημένα

10 (i) γρ. εξαρτημένα (ii) γρ. εξαρτημένα (iii) γρ. εξαρτημένα

11 $\dim W = 3$

12 (i) Όχι (ii) Ναι (iii) Όχι (iv) Όχι

13 (i) $k \neq -1$ (ii) $k \neq -1$

15

16 $(-1, 1, -1, 3)$

17 $(4, -3, 1)$

18 $(0, 2, -1)$

19 (i) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 7 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

20 (i) $(1, 1, -4, -3),\ (0, 1, -5, -2),\ (0, 0, 1, -1/2)$
(ii) $(1, -1, 2, 0),\ (0, 1, 0, 0),\ (0, 0, 1, -1/6)$
(iii) $(1, 1, 0, 0),\ (0, 1, 1, 1),\ (0, 0, 1, 1),\ (0, 0, 0, 1)$

21 (i) $\{\mathbf{u}_1, \mathbf{u}_2\},\ \mathbf{u}_3 = 2\mathbf{u}_1 + \mathbf{u}_2,\ \mathbf{u}_4 = -2\mathbf{u}_1 + \mathbf{u}_2$
(ii) $\{\mathbf{u}_1, \mathbf{u}_3\},\ \mathbf{u}_2 = 2\mathbf{u}_1,\ \mathbf{u}_4 = \mathbf{u}_1 + \mathbf{u}_3$
(iii) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_4\},\ \mathbf{u}_3 = 2\mathbf{u}_1 - \mathbf{u}_2,\ \mathbf{u}_5 = -\mathbf{u}_1 + 3\mathbf{u}_2 + 2\mathbf{u}_4$

22 (i) rank=3, nullity=3 (ii) rank=4, nullity=1 (iii) rank=2, nullity=4

Κεφάλαιο 6

1 (i) $\lambda = 1,\ [0\ 1\ 0]^T,\ \ \lambda = 2,\ [-1\ 2\ 2]^T,\ \ \lambda = 3,\ [-1\ 1\ 1]^T$
(ii) $\lambda = 0,\ [5\ 1\ 3]^T,\ \ \lambda = \sqrt{2},\ [\frac{1}{7}(15 + 5\sqrt{2})\ \frac{1}{7}(-1 + 2\sqrt{2})\ 1]^T,\ \ \lambda = -\sqrt{2},\ [\frac{1}{7}(15 - 5\sqrt{2})\ \frac{1}{7}(-1 - 2\sqrt{2})\ 1]^T$
(iii) $\lambda = -8,\ [-\frac{1}{6}\ -\frac{1}{6}\ 1]^T$
(iv) $\lambda = 2,\ [1\ 1\ 3]^T$
(v) $\lambda = 2,\ [-1\ -1\ 3]^T$
(vi) $\lambda = -4,\ [-2\ \frac{8}{3}\ 1]^T,\ \ \lambda = 3,\ [5\ -2\ 1]^T$

2 $\lambda = 1,\ \lambda = 512,\ \lambda = \frac{1}{512},\ \lambda = 0$

3 $\lambda = 1,\ [-1\ 1\ 0]^T,\ [-1\ 0\ 1]^T$ και $\lambda = -1,\ [2\ -1\ 1]^T$

4 (i) Όχι (ii) Ναι, $\mathbf{P} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (iii) Όχι

(iv) Ναι, $\mathbf{P} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ (v) Όχι (vi) Ναι, $\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5 (i) $\mathbf{A}^{49} = \mathbf{A}$ (ii) $\begin{bmatrix} 4188163 & 6282243 & -9421830 \\ 4192254 & 6288382 & -9432060 \\ 4190208 & 6285312 & -9426944 \end{bmatrix}$

6 Μια πιθανή απάντηση: $\begin{bmatrix} 3 & -2 & -2 \\ -7 & 10 & 11 \\ 8 & -10 & -11 \end{bmatrix}$

7 (i) $\mathbf{P} = \begin{bmatrix} -\frac{4}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$, $\mathbf{P}^{-1}\mathbf{AP} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -50 \end{bmatrix}$

(ii) $\mathbf{P} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$, $\mathbf{P}^{-1}\mathbf{AP} =$

(iii) $\mathbf{P} = \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} & 0 & 0 \\ \frac{3}{5} & \frac{4}{5} & 0 & 0 \\ 0 & 0 & -\frac{4}{5} & \frac{3}{5} \\ 0 & 0 & \frac{3}{5} & \frac{4}{5} \end{bmatrix}$, $\mathbf{P}^{-1}\mathbf{AP} = \begin{bmatrix} -25 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & -25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix}$

8 $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$