Εισαγωγή

2. (α) Δεν υπάρχουν κρίσιμα σημεία.
(β) Σχετικό ελάχιστο: $(1,2)$.
(γ) Σαγματικό σημείο: $(n\pi, 0), \quad n = 0, \pm1, \pm2, ...$

3. (α) $(1,2,2)$
(β) $(\pm\sqrt{5}, 0, 0)$

4. (α) (i) Ελάχιστο: $f\left(-\frac{4}{3}, -\frac{2}{3}, \frac{4}{3}\right) = -6$ Μέγιστο: $f\left(\frac{4}{3}, \frac{2}{3}, -\frac{4}{3}\right) = 6$

(ii) Μέγιστο $= \frac{1}{3\sqrt{3}}$ στα σημεία $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Ελάχιστο $= -\frac{1}{3\sqrt{3}}$ στα σημεία $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

(β) $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{6}\right)$

7. $u = F(x) + \frac{1}{x}G(y)$

$u = x^5 + x - 68\frac{1}{x} + 6\frac{y^4}{x}$

8. $u = F(2x+y) + G(2x-y) + \frac{1}{4}x\mathrm{e}^{2x+y}$

9. $u_x = \frac{1-12v}{1-8uv},\ v_x = \frac{2u-3}{1-8uv},\ u_y = \frac{2+4v}{8uv-1},\ v_y = \frac{4u+1}{8uv-1}$

10. $\frac{\mathrm{d}^2F}{\mathrm{d}\eta^2} + \left(\frac{\mathrm{d}F}{\mathrm{d}\eta}\right)^2 + \frac{1}{2}\eta\frac{\mathrm{d}F}{\mathrm{d}\eta} = 0, \quad \eta = \frac{x}{\sqrt{t}}$

11. (i) $u(x,t) = 10\mathrm{e}^{-x-3t} - 6\mathrm{e}^{-4x-6t}$
(ii) $u(x,t) = 6\mathrm{e}^{-\frac{\pi^2}{4}t}\sin\left(\frac{\pi}{2}x\right) + 3\mathrm{e}^{-\pi^2 t}\sin\pi x$

Κεφάλαιο 1

1. (α) $y(x) = -\frac{\sinh(x-2)}{\sinh 1}$
(β) $y = x, \quad \mathbf{I} = \frac{1}{2}\pi\left(1 + \frac{1}{12}\pi^2\right)$
(γ) $y = c_1 \sin x$

(δ) $y=\frac{2x}{x+1}$

2. $y^2+(x-c_1)^2=c_2$

3. $y=c_1\cosh\left(\dfrac{x+c_2}{c_1}\right)$

4. $\rho\cos(\phi+c_1)=c_2$

7. $y=\sqrt{\dfrac{2}{\pi}}\sin(nx),\quad n=1,2,3,...$

8. (α) $y=1-\cos x$

(β) $y=\dfrac{1}{c_1}\cosh(c_1x+c_2)+\lambda$

9. $y=\dfrac{1}{4}\dfrac{\lambda}{\lambda+1}(x^2-x)+x,\quad z=x$

10. $y=c_2\mathrm{e}^{x}+c_3\mathrm{e}^{-x}+\frac{1}{4}c_1,\quad z=-2c_3\mathrm{e}^{-x}+\frac{1}{4}c_1x+c_4$

17. $(m_1+m_2)l_1\ddot{\theta}_1+m_2l_2(\ddot{\theta}_2\cos\alpha+\dot{\theta}_2^2\sin\alpha)+(m_1+m_2)g\sin\theta_1=0,$

$l_1\ddot{\theta}_1\cos\alpha+l_2\ddot{\theta}_2-l_1\dot{\theta}_1^2\sin\alpha+g\sin\theta_2=0,$

όπου $\alpha=\theta_1-\theta_2$

18. $m[\ddot{r}-r\dot{\theta}^2-r\sin^2\theta\dot{\phi}^2]=-V_r,$

$m\left[\dfrac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})-r^2\sin\theta\cos\theta\dot{\phi}^2\right]=-V_\theta,$

$m\dfrac{\mathrm{d}}{\mathrm{d}t}\left[r^2\sin^2\theta\dot{\phi}\right]=-V_\phi.$

19. $\mathbf{I}[y(x)]=\displaystyle\int_0^1\left(y'^2-y^2-2xy\right)\mathrm{d}x$

$y=\frac{5}{18}x(1-x)$

$y=\frac{71}{369}x(1-x)+\frac{7}{41}x^2(1-x)$

20. $y=\frac{21}{28-2\pi^2}x^2(\pi-x)$

21. $\mathbf{I}[y(x)]=\displaystyle\int_0^1\left[(1-x^2)y'^2-12y^2\right]\mathrm{d}x$

Ακριβής λύση: $y=\frac{1}{2}(5x^3-3x)$

22. $c_1=\dfrac{85}{26},\quad c_2=-\dfrac{35}{13}$

24. (α) $\lambda=15$

(β) $\lambda=14.42$ ή $\lambda=63.58$

Κεφάλαιο 2

1. (α) $\dfrac{1}{s^2}\left(1-\mathrm{e}^{-s}\right)$ (β) $\dfrac{1+\mathrm{e}^{-s\pi}}{1+s^2}$ (γ) $\dfrac{1}{s^2+2s+2}$

(δ) $\dfrac{s^2-1}{(s^2+1)^2}$ (ε) $\dfrac{8}{s^3}-\dfrac{15}{s^2+9}$ (στ) $\dfrac{1}{2(s-2)}-\dfrac{1}{2s}$

(ζ) $\dfrac{2}{s^2+16}$ (η) $\dfrac{1}{2}\left(\dfrac{3}{s^2+9}-\dfrac{1}{s^2+1}\right)$

2. (α) $\sqrt{\dfrac{\pi}{s}}$ (β) $\dfrac{\sqrt{\pi}}{2s^{3/2}}$ (γ) $\dfrac{3}{4}\dfrac{\sqrt{\pi}}{s^{5/2}}$

4. (α) $\dfrac{6}{(s+2)^4}$ (β) $\dfrac{1}{(s-2)^2}+\dfrac{2}{(s-3)^2}+\dfrac{1}{(s-4)^4}$

(γ) $\dfrac{\mathrm{e}^{-2s}}{s^2}+2\dfrac{\mathrm{e}^{-2s}}{s}$ (δ) $\dfrac{s}{s^2+4}\mathrm{e}^{-\pi s}$

6.(α) $f(t)=1-\mathcal{U}(t-4)+\mathcal{U}(t-5),\quad \mathcal{L}\{f(t)\}=\dfrac{1}{s}\left(1-\mathrm{e}^{-4s}+\mathrm{e}^{-5s}\right)$

(β) $f(t)=t^2\mathcal{U}(t-1),\quad \mathcal{L}\{f(t)\}=2\dfrac{\mathrm{e}^{-s}}{s^3}+2\dfrac{\mathrm{e}^{-s}}{s^2}+\dfrac{\mathrm{e}^{-s}}{s}$

(γ) $f(t)=t-t\mathcal{U}(t-2),\quad \mathcal{L}\{f(t)\}=\dfrac{1}{s^2}-\dfrac{\mathrm{e}^{-2s}}{s^2}-2\dfrac{\mathrm{e}^{-2s}}{s}$

(δ) $f(t)=\sin t-\sin t\,\mathcal{U}(t-2\pi),\quad \mathcal{L}\{f(t)\}=\dfrac{1-\mathrm{e}^{-2\pi s}}{s^2+1}$

7. (α) $f(t)=\left|\sin t\right|,\ \ f(t+\pi)=f(t),\ \ \mathcal{L}\{f(t)\}=\dfrac{\coth(\pi s/2)}{s^2+1}$

(β) $f(t)=\begin{cases}\sin t, & 0<t\le\pi\\ 0, & \pi<t\le 2\pi\end{cases},\ \ f(t+2\pi)=f(t),\ \ \mathcal{L}\{f(t)\}=\dfrac{1}{(1-\mathrm{e}^{-\pi s})(s^2+1)}$

8. $\dfrac{1-(s+1)\mathrm{e}^{-s}}{s^2(1-\mathrm{e}^{-2s})}$

10. (α) $\dfrac{1}{s(s-1)}$ (β) $\dfrac{s+1}{s\left[(s+1)^2+1\right]}$ (γ) $\dfrac{1}{s^2(s-1)}$

11. (α) $\dfrac{6}{s^5}$ (β) $\dfrac{48}{s^8}$ (γ) $\dfrac{s-1}{(s+1)\left[(s-1)^2+1\right]}$

12. (α) $\dfrac{s^2-a^2}{(s^2+a^2)^2}$ (β) $\dfrac{6s^2-2}{(s^2+1)^3}$ (γ) $\dfrac{6s^4-36s^2+6}{(s^2+1)^4}$ (δ) $\ln\left(\dfrac{s+b}{s+a}\right)$

(ε) $\dfrac{1}{2}\ln\left(\dfrac{s+1}{s-1}\right)$

14. (α) $1+3t+\frac{3}{2}t^2+\frac{1}{6}t^3$ (β) $\frac{1}{4}\mathrm{e}^{-t/4}$ (γ) $\frac{1}{4}\sinh 4t$

(δ) $2\cos 3t-2\sin 3t$ (ε) $\frac{1}{2}\mathrm{e}^{2t}-\mathrm{e}^{3t}+\frac{1}{2}\mathrm{e}^{6t}$ (στ) $\frac{1}{3}\sin t-\frac{1}{6}\sin 2t$

15. (α) $\frac{1}{2}t^2\mathrm{e}^{-2t}$ (β) $\mathrm{e}^{-t}(1-t)$

(γ) $5-t-\mathrm{e}^{-t}\left(5+4t+\frac{3}{2}t^2\right)$ (δ) $\frac{1}{2}(t-2)^2\mathcal{U}(t-2)$

(ε) $\mathcal{U}(t-1)\left(1-\mathrm{e}^{-(t-1)}\right)$ (στ) $\frac{1}{2}t\sin t$

16. (α) $1-\mathrm{e}^{-t}$ (β) $-\frac{1}{3}\mathrm{e}^{-t}+\frac{1}{3}\mathrm{e}^{2t}$
(γ) $\frac{1}{4}t\sin 2t$ (δ) $\frac{1}{4}\sin t\mathrm{e}^{-2t}-\frac{1}{2}t\mathrm{e}^{-2t}\cos t+\frac{1}{4}\mathrm{e}^{-2t}\sin t$

18. (α) $\sqrt{\dfrac{\pi}{s+2}}$ (β) $\dfrac{\ln(s^2+1)}{s^3}-\dfrac{3s^2+1}{s(s^2+1)^2}$ (γ) $\dfrac{1}{s+1}\tan^{-1}\left(\dfrac{2}{s+1}\right)$
(δ) $\dfrac{1}{(s-3)\sqrt{s-2}}$ (ε) $\dfrac{\sqrt{s+1}-1}{s\sqrt{s+1}}$ (στ) $\dfrac{1}{s^2\sqrt{s+1}}$
(ζ) $\dfrac{2}{s^3}\mathrm{e}^{-2s}(1+2s+2s^2)$ (η) $\frac{1}{4}\ln\left(\dfrac{s^2+4}{s^2}\right)$ (θ) $\dfrac{\mathrm{e}^{-s}}{s^2}(s^2+s+1)$

21. (α) $\frac{1}{2}\mathrm{e}^{t}-\mathrm{e}^{2t}+\frac{5}{2}\mathrm{e}^{3t}$ (β) $\frac{1}{5}\mathrm{e}^{-t}(4\cos t-3\sin t)-\frac{4}{5}\mathrm{e}^{-3t}$ (γ) $\frac{1}{2}\sin t\sinh t$

22. (α) $\mathrm{e}^t\mathrm{erf}\sqrt{t}+\dfrac{1}{\sqrt{\pi t}}$ (β) $(1-\cos(t-1))\mathcal{U}(t-1)-(1-\cos(t-2))\mathcal{U}(t-2)$
(γ) $\mathrm{e}^t\mathrm{erf}\sqrt{t}+\dfrac{1}{\sqrt{\pi t}}-\mathrm{e}^t$

23. (α) $y(t)=\frac{1}{9}t+\frac{2}{27}-\frac{2}{27}\mathrm{e}^{3t}+\frac{10}{9}t\mathrm{e}^{3t}$
(β) $y(t)=\frac{1}{2}\left(\mathrm{e}^t\sin t-\mathrm{e}^t\cos t+1\right)$
(γ) $y(t)=-t+\frac{1}{2}\sin t+\frac{1}{4}\mathrm{e}^t-\frac{1}{4}\mathrm{e}^{-t}$
(δ) $y(t)=\cos 2t-\frac{1}{6}\sin 2(t-2\pi)\mathcal{U}(t-2\pi)+\frac{1}{3}\sin(t-2\pi)\mathcal{U}(t-2\pi)$
(ε) $y(t)=\mathrm{e}^{-2t}\cos 3t+\frac{2}{3}\mathrm{e}^{-2t}\sin 3t+\frac{1}{3}\mathrm{e}^{-2(t-\pi)}\sin 3(t-\pi)\mathcal{U}(t-\pi)$
$+\frac{1}{3}\mathrm{e}^{-2(t-3\pi)}\sin 3(t-3\pi)\mathcal{U}(t-3\pi)$

24. (α) $y(t)=(\mathrm{e}-1)\mathrm{e}^{-t}+(\mathrm{e}+1)t\mathrm{e}^{-t}$
(β) $y(t)=\frac{1}{3}t^3+ct^2$

25. (α) $x(t)=-\frac{1}{2}t-\frac{3}{4}\sqrt{2}\sin(\sqrt{2}t)$, $y(t)=-\frac{1}{2}t+\frac{3}{4}\sqrt{2}\sin(\sqrt{2}t)$
(β) $x(t)=1+t+\frac{1}{2}t^2-\mathrm{e}^{-t}$, $y(t)=-\frac{1}{3}(1-\mathrm{e}^{-t}-t\mathrm{e}^{-t})$

26. $u(x,t)=u_0\mathrm{erfc}\left(\frac{x}{2\sqrt{t}}\right)$

Κεφάλαιο 3

4. $c_1=2,\ c_2=-1,\ c_3=\frac{2}{3},\quad E^2=\dfrac{18\pi^3-49}{18\pi}$

5. (α) $f(x)=\pi+2\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$
(β) $f(x)=\dfrac{2\sinh\pi}{\pi}\left[\dfrac{1}{2}+\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}(\cos nx-n\sin nx)\right]$
(γ) $f(x)=\dfrac{1}{\pi}+\dfrac{1}{2}\sin x+\dfrac{1}{\pi}\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n+1}{1-n^2}\cos nx$

(δ) $f(x) = -\dfrac{1}{4} + \dfrac{1}{\pi}\sum_{n=1}^{\infty}\left[-\dfrac{1}{n}\sin\dfrac{n\pi}{2}\cos\dfrac{n\pi}{2}x + \dfrac{3}{n}\left(1-\cos\dfrac{n\pi}{2}\right)\sin\dfrac{n\pi}{2}x\right]$

(ε) $f(x) = \dfrac{9}{4} + 5\sum_{n=1}^{\infty}\left[\dfrac{(-1)^n-1}{n^2\pi^2}\cos\dfrac{n\pi}{5}x + \dfrac{(-1)^{n+1}}{n\pi}\sin\dfrac{n\pi}{5}x\right]$

(στ) $f(x) = \dfrac{1}{\pi}(\mathrm{e}^{\pi}-\pi-1) + \dfrac{1}{\pi}\sum_{n=1}^{\infty}\left[\dfrac{(-1)^n\mathrm{e}^{\pi}-1}{n^2+1}\cos nx + \dfrac{(-1)^n(1+n^2-n^2\mathrm{e}^{\pi})-1}{n(n^2+1)}\sin nx\right]$

10. (α) $f(x) = \dfrac{\pi}{2} + \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n-1}{n^2}\cos nx$

(β) $f(x) = \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{1-(-1)^n(1+\pi)}{n}\sin nx$

(γ) $f(x) = \dfrac{2}{\pi}\left(1 + \sum_{n=2}^{\infty}\dfrac{1+(-1)^n}{1-n^2}\cos nx\right)$

(δ) $f(x) = \dfrac{3}{4} + \dfrac{4}{\pi^2}\sum_{n=1}^{\infty}\dfrac{\cos\dfrac{n\pi}{2}-1}{n^2}\cos\dfrac{n\pi}{2}x$

11. (α) $f(x) = \dfrac{2}{\pi} + \dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{1-4n^2}\cos 2nx$

$f(x) = \dfrac{8}{\pi}\sum_{n=1}^{\infty}\dfrac{n}{4n^2-1}\sin 2nx$

(β) $f(x) = \dfrac{5}{6} + \dfrac{2}{\pi^2}\sum_{n=1}^{\infty}\dfrac{3(-1)^n-1}{n^2}\cos n\pi x$

$f(x) = 4\sum_{n=1}^{\infty}\left[\dfrac{(-1)^{n+1}}{n\pi} + \dfrac{(-1)^n-1}{n^3\pi^3}\right]\sin n\pi x$

(γ) $f(x) = \dfrac{\pi}{4} + \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{2\cos\dfrac{n\pi}{2}-(-1)^n-1}{n^2}\cos nx$

$f(x) = \dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{\sin\dfrac{n\pi}{2}}{n^2}\sin nx$

(δ) $f(x) = \dfrac{3}{4} + \dfrac{4}{\pi^2}\sum_{n=1}^{\infty}\dfrac{\cos\dfrac{n\pi}{2}-1}{n^2}\cos\dfrac{n\pi}{2}x$

$f(x) = \sum_{n=1}^{\infty}\left[\dfrac{4}{n^2\pi^2}\sin\dfrac{n\pi}{2} - \dfrac{2}{n\pi}(-1)^n\right]\sin\dfrac{n\pi}{2}x$

12. (α) $f(x) = \dfrac{4\pi^2}{3} + 4\sum_{n=1}^{\infty}\left[\dfrac{1}{n^2}\cos nx - \dfrac{\pi}{n}\sin nx\right]$

(β) $f(x) = \dfrac{3}{2} - \dfrac{1}{\pi}\sum_{n=1}^{\infty} \dfrac{1}{n}\sin 2n\pi x$

13. $f(x) = \dfrac{1}{4} + \dfrac{2}{\pi^2}\sum_{m=1}^{\infty}\dfrac{(-1)^m - 1}{m^2}\cos m\pi x + \dfrac{2}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n - 1}{n^2}\sin n\pi y$

$+\dfrac{4}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\dfrac{((-1)^m - 1)((-1)^n - 1)}{m^2 n^2}\cos m\pi x \sin n\pi y$

20. (α) $f(x) = \dfrac{1}{\pi}\int_0^{\infty}\left[\dfrac{3\alpha\sin 3\alpha + \cos 3\alpha - 1}{\alpha^2}\cos\alpha x + \dfrac{\sin 3\alpha - 3\alpha\cos 3\alpha}{\alpha^2}\sin\alpha x\right]\mathrm{d}\alpha$

(β) $f(x) = \dfrac{1}{\pi}\int_0^{\infty}\dfrac{\cos\alpha x + \alpha\sin\alpha x}{1+\alpha^2}\mathrm{d}\alpha$

21. (α) $f(x) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\pi\alpha\sin\pi\alpha + \cos\pi\alpha - 1}{\alpha^2}\cos\alpha x\mathrm{d}\alpha$

(β) $f(x) = \dfrac{4}{\pi}\int_0^{\infty}\dfrac{\alpha\sin\alpha x}{\alpha^4 + 4}\mathrm{d}\alpha$

22. (α) $f(x) = \dfrac{2k}{\pi}\int_0^{\infty}\dfrac{\cos\alpha x}{k^2+\alpha^2}\mathrm{d}\alpha, \quad f(x) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\alpha\sin\alpha x}{k^2+\alpha^2}\mathrm{d}\alpha$

(β) $f(x) = \int_0^{\infty}\dfrac{4-\alpha^2}{(4+\alpha^2)^2}\cos\alpha x\mathrm{d}\alpha, \quad f(x) = \dfrac{8}{\pi}\int_0^{\infty}\dfrac{\alpha}{(4+\alpha^2)^2}\sin\alpha x\mathrm{d}\alpha$

23. $f(x) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\sin\alpha\cos\alpha(x-1)}{\alpha}\mathrm{d}\alpha$

24. $F(\alpha) = \dfrac{4}{\alpha^3}(\alpha\cos\alpha - \sin\alpha)$

25. $F(\alpha) = \dfrac{\alpha}{1+\alpha^2}$

26. $\mathcal{F}_s\{f(x)\} = \dfrac{1-\cos\alpha}{\alpha}, \quad \mathcal{F}_c\{f(x)\} = \dfrac{\sin\alpha}{\alpha}$

27. (α) $u(x,y) = \dfrac{2}{\pi}\int_0^{\infty}F(\alpha)\dfrac{\sinh\alpha(2-y)}{\sinh 2\alpha}\sin\alpha x\mathrm{d}\alpha$

(β) $u(x,y) = \dfrac{2}{\pi}\int_0^{\infty}\dfrac{\sinh\alpha(\pi - x)}{(1+\alpha^2)\sinh\alpha\pi}\cos\alpha y\mathrm{d}\alpha$

29. (α) $u(x,t) = \mathrm{e}^{-\gamma^2 kt}\left(a_0 + \sum_{n=1}^{\infty}a_n\cos\left(\dfrac{n\pi x}{L}\right)\exp\left(-\dfrac{n^2\pi^2 kt}{L^2}\right)\right),$

$a_0 = \dfrac{1}{2}, \; a_n = \dfrac{2(1-\cos(n\pi))}{n^2\pi^2}$

(β) $u(x,t) = \dfrac{L^2}{\pi^3}\sum_{n=1}^{\infty}\dfrac{1-(-1)^n}{n^3}\cos\dfrac{n\pi\alpha t}{L}\sin\dfrac{n\pi x}{L}$

30. $u(x,t)=\dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{(2n-1)^2}\sin(2n-1)x\cos\sqrt{(2n-1)^2+1}t$

Κεφάλαιο 4

1. (α) $y_1(x)=c_0\sum_{n=1}^{\infty}\dfrac{1}{n}x^n,\quad y_2(x)=0$

(β) $y_1(x)=c_0\sum_{n=1}^{\infty}x^{2n},\quad y_2(x)=c_1\sum_{n=0}^{\infty}x^{2n+1}$

(γ) $y_1(x)=c_0\left(1+\dfrac{1}{4}x^2-\dfrac{7}{4.4!}x^4+\dfrac{23.7}{8.6!}x^6-...\right)$

$y_2(x)=c_1\left(x-\dfrac{1}{6}x^3+\dfrac{14}{2.5!}x^5-\dfrac{34.14}{4.7!}x^7+...\right)$

(δ) $y_1(x)=c_0\left(1+\frac{1}{2}x+\frac{1}{6}x^3+\frac{1}{6}x^4+...\right),\quad y_2(x)=c_1\left(x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{1}{4}x^4+...\right)$

2. (α) $y(x)=8x-2\mathrm{e}^x$

(β) $y(x)=3-12x^2+4x^4$

3 (α) $y(x)=c_1x^{\frac{3}{2}}\left(1-\dfrac{2}{5}x+\dfrac{2^2}{7.5.2}x^2-\dfrac{2^3}{9.7.5.3!}x^3+...\right)+c_2\left(1+2x-2x^2+\dfrac{2^3}{3.3!}x^3-...\right)$

(β) $y(x)=c_1x^{\frac{1}{3}}\left(1+\dfrac{1}{3}x+\dfrac{1}{3^2.2}x^2+\dfrac{1}{3^3.3!}x^3+...\right)+c_2\left(1+\dfrac{1}{2}x+\dfrac{1}{5.2}x^2+\dfrac{1}{8.5.2}x^3+...\right)$

(γ) $y(x)=c_1x\left(1+\dfrac{1}{5}x+\dfrac{1}{5.7}x^2+\dfrac{1}{5.7.9}x^3+...\right)+c_2x^{-\frac{1}{2}}\left(1+\dfrac{1}{2}x+\dfrac{1}{2.4}x^2+\dfrac{1}{2.4.6}x^3+...\right)$

(δ) $y(x)=\dfrac{1}{x}\left(c_1\cosh x+c_2\sinh x\right)$

(ε) $y(x)=c_1\mathrm{e}^x+c_2\mathrm{e}^x\left(\ln x-x+\dfrac{1}{2.2!}x^2-\dfrac{1}{3.3!}x^3+\dfrac{1}{4.4!}x^4-...\right)$

(στ) $y(x)=c_1x^2+c_2\left(\dfrac{1}{2}x^2\ln x-\dfrac{1}{2}+x-\dfrac{1}{3!}x^3+...\right)$

4. (α) $y(x)=c_1J_{\frac{1}{3}}(x)+c_2J_{-\frac{1}{3}}(x)$

(β) $y(x)=c_1J_{\frac{5}{2}}(x)+c_2J_{-\frac{5}{2}}(x)$

(γ) $y(x)=c_1J_0(x)+c_2Y_0(x)$

(δ) $y(x)=c_1J_2(3x)+c_2Y_2(3x)$

5. $y(x)=\dfrac{c_1}{\sqrt{x}}J_{\frac{1}{2}}(x)+\dfrac{c_2}{\sqrt{x}}J_{-\frac{1}{2}}(x)$

8. (α) $J_{\frac{3}{2}}(x)=\sqrt{\dfrac{2}{\pi x}}\left(\dfrac{\sin x-x\cos x}{x}\right)$ (β) $J_{-\frac{3}{2}}(x)=-\sqrt{\dfrac{2}{\pi x}}\left(\dfrac{x\sin x+\cos x}{x}\right)$

(γ) $J_{\frac{5}{2}}(x)=\sqrt{\dfrac{2}{\pi x}}\left[\left(\dfrac{3}{x^2}-1\right)\sin x-\dfrac{3}{x}\cos x\right]$ (δ) $J_{-\frac{5}{2}}(x)=\sqrt{\dfrac{2}{\pi x}}\left[\dfrac{3}{x}\sin x+\left(\dfrac{3}{x^2}-1\right)\cos x\right]$

10. $J_n^{(k)}(x) = \frac{1}{2^k}\left[J_{n-k}(x) - \begin{pmatrix} k \\ k-1 \end{pmatrix} J_{n-k+2} + \begin{pmatrix} k \\ k-2 \end{pmatrix} J_{n-k+4} - ...\right]$

11. (α) $x^3 J_3(x) + c$

(β) $J_1(1) - 2J_2(1)$

(γ) $-\frac{1}{3}\left(\frac{J_2(x)}{x} + J_1(x) - \int J_0(x)\mathrm{d}x\right) + c$

Κεφάλαιο 5

1. $\phi(x) = \frac{1}{2}\mathrm{e}^{-x} - \frac{1}{2}\int_0^x \phi(s)\mathrm{d}s, \quad \phi'(x) + \frac{1}{2}\phi(x) = -\frac{1}{2}\mathrm{e}^{-x}, \quad \phi(x) = -\frac{1}{2}\mathrm{e}^{-\frac{x}{2}} + \mathrm{e}^{-x}$

2. (α) $\phi(x) = x + \frac{1}{2}\frac{\lambda}{1-\lambda}, \quad \lambda_1 = 1$

(β) $\phi(x) = 1 + \frac{(n+m+1)\lambda x^n}{n+m+1-\lambda)(m+1)}, \quad \lambda_1 = n+m+1$

3. (α) $\lambda_1 = -\frac{3}{4}, \quad \phi_1(x) = Ax$ $\qquad \lambda_{2,3} = \frac{3\sqrt{5}}{2\sqrt{5} \pm 6}, \quad \phi_{2,3}(x) = A\left(1 \pm \sqrt{5}x^2\right)$

(β) $\lambda_1 = \frac{1}{\pi}, \quad \phi_1(x) = A$ $\qquad \lambda_2 = \frac{2}{\pi}, \quad \phi_2(x) = A\cos 2x$

$\lambda_3 = -\frac{2}{\pi}, \quad \phi_3(x) = A\sin 2x$

4. (α) $\phi(x) = \mathrm{e}^x$ $\qquad$ (β) $\phi(x) = \mathrm{e}^{x^2}$ $\qquad$ (γ) $\phi(x) = \frac{n}{n-1}x^{n-1}, \; n \neq 1$

(δ) $\phi(x) = \cosh x - x\mathrm{e}^{-x}$

7. (α) $y(x) = \frac{1}{6}(x^3 - 3x) - \int_0^1 G(x,s)y(s)\mathrm{d}s, \quad G(x,s) = \begin{cases} -x, & x \le s \\ -s, & x \ge s \end{cases}$

(β) $y(x) = \frac{1}{6}(x^3 - 3x + 6) - \int_0^1 G(x,s)y(s)\mathrm{d}s, \quad G(x,s) = \begin{cases} -x, & x \le s \\ -s, & x \ge s \end{cases}$

8. $y(x) = -\lambda \int_0^1 G(x,s)sy(s)\mathrm{d}s$

10. (α) $\phi(x) = \sin x$ $\qquad$ (β) $\phi(x) = -\frac{1}{8}\mathrm{e}^{-x} + \left(\frac{1}{8} + \frac{3}{4}x + \frac{1}{4}x^2\right)\mathrm{e}^x$

(γ) $\phi(x) = \mathrm{e}^{-x}$ $\qquad$ (δ) $\phi(x) = \frac{3}{8}\mathrm{e}^{2x} + \frac{1}{8}\mathrm{e}^{-2x} + \frac{1}{2}\cos 2x + \frac{1}{4}\sin 2x$

11.(α) $\phi(x) = x + \frac{1}{6}x^3$ $\qquad$ (β) $\phi(x) = \cosh x - x\mathrm{e}^{-x}$

13. $\phi(x) = f'(x) - f(x)$

14. $\phi(x) = \frac{1}{1-\lambda^2}f(x) + \frac{\lambda}{1-\lambda^2}\sqrt{\frac{2}{\pi}}\int_0^{+\infty} \cos(xs)f(s)\mathrm{d}s, \quad |\lambda| \neq 1$